Getting Ready to Teach Unit 4

Learning Path in the Common Core Standards

In this unit, students multiply whole numbers and decimals. Their work involves finding the products of two-digit whole numbers and decimals both less than 1 and greater than 1, and estimating products by rounding factors.

Visual models and real world situations are used throughout the unit to illustrate important number and operation concepts.

Math Expressions VOCABULARY

As you teach this unit, emphasize understanding of these terms:

- shift patterns
- Place Value Rows
- Place Value Sections
- New Groups Above
- New Groups Below
- Short Cut Method

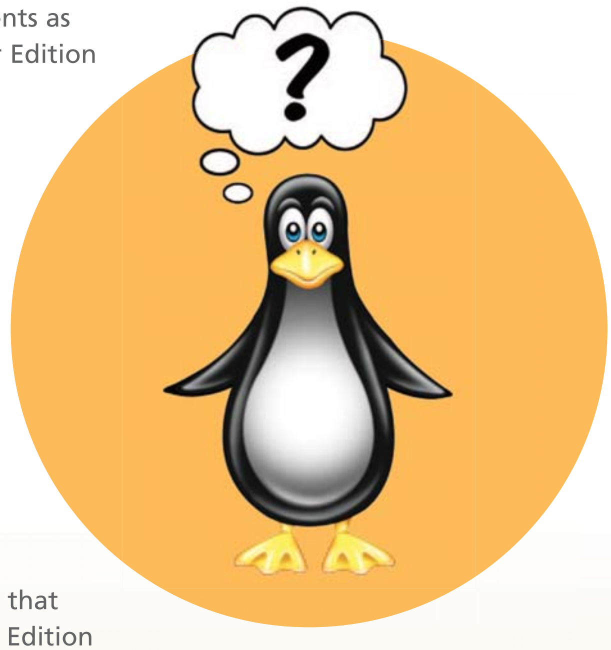
See the Teacher Glossary

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, and explaining why this way is correct and why the error is wrong. The following common errors are presented to students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin:

- **Lesson 2:** Assuming the number of zeros in a product is always the same as the number of zeros in the factors
- **Lesson 4:** Regrouping incorrectly when finding a product
- **Lesson 7:** Shifting the digits in the wrong direction
- **Lesson 8:** Unable to apply properties to simplify finding products
- **Lesson 9:** Not using zero as a placeholder and using an exponent as a factor
- **Lesson 10:** Rounding cents incorrectly



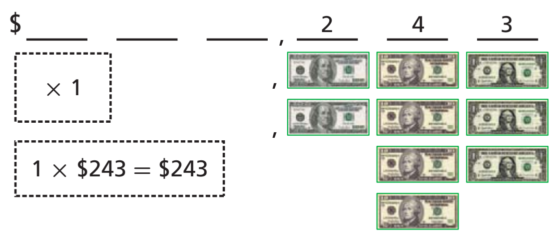
In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

Shift Patterns in Multiplication

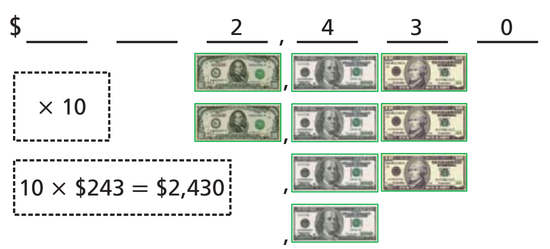
Lessons 1 2

Multiplying Whole Numbers by Powers of Ten Real world scenarios involving money are used to introduce students to the concept of shifting digits. For whole numbers, Jordan's salary is given as $243 per week, and students consider the amount earned in 1, 10, 100, and 1,000 weeks.

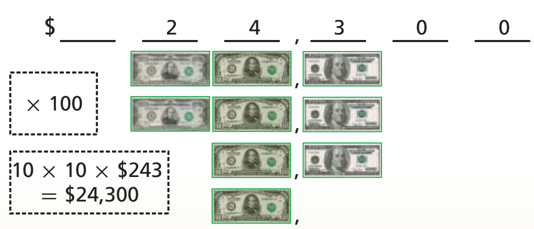
Jordan's Weekly Earnings



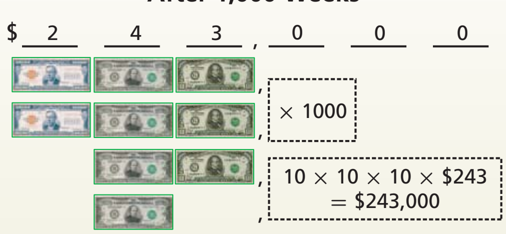
After 10 Weeks



After 100 Weeks



After 1,000 Weeks



Jordan's earnings show students that the result of multiplying by 1, 10, 100 and 1,000 shifts the digits in the multiplicand to the left. Multiplying by 10 shifts the digits one place to the left, by 100 shifts the digits two places, and by 1,000 shifts the digits three places.

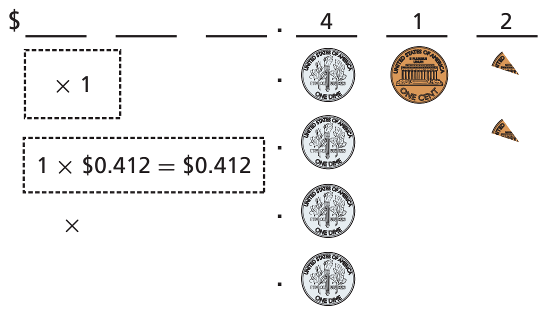
***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN**

Place Value and Shift Patterns
Students extend their understanding of the base-ten system to the relationship between adjacent places, how numbers compare, and how numbers round for decimals to thousandths.

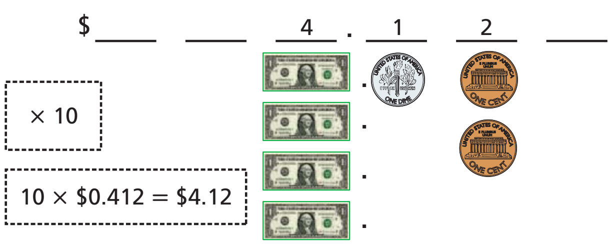
New at Grade 5 is the use of whole number exponents to denote powers of 10. Students understand why multiplying by a power of 10 shifts the digits of a whole number or decimal that many places to the left. For example, multiplying by 10^4 is multiplying by 10 four times. Multiplying by 10 once shifts every digit of the multiplicand one place to the left in the product (the product is ten times as large) because in the base-ten system the value of each place is 10 times the value of the place to its right. So multiplying by 10 four times shifts every digit 4 places to the left. Patterns in the number of 0s in products of whole numbers and a power of 10, and the location of the decimal point in products of decimals with powers of 10, can be explained in terms of place value.

Multiplying Decimals by Powers of Ten For decimals, the same concept is presented by giving the manufacturing cost of an item as \$0.412, and having students consider the cost for the production of 1, 10, 100, and 1,000 items.

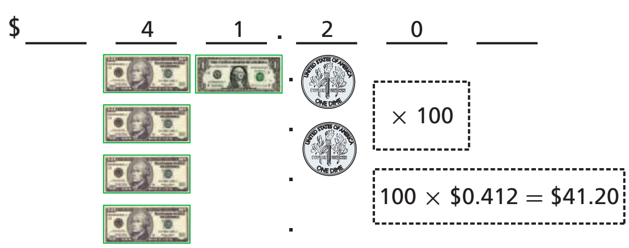
Cost of a Red Phantom Marble



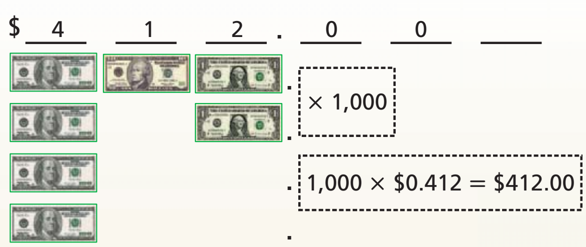
10 Red Phantom Marbles



100 Red Phantom Marbles



1,000 Red Phantom Marbles



Again students see that multiplying by 10, 100, and 1,000 shifts the digits in the multiplicand to the left. Multiplying by 10 shifts the digits one place to the left, by 100 shifts the digits two places, and by 1,000 shifts the digits three places.

Patterns in Multiplying with Zeros Exploring the Shift Pattern of zeros in the table below enables students to generalize that shifting digits any number of places to the left shifts the decimal point the same number of places to the right. It also gives students a way to predict the number of zeros in a product.

$\times$	3	30	300	3,000
2	a. 2×3 $= 6$	b. 2×30 $= 2 \times 3 \times 10$ $= 6 \times 10$ $= 60$	c. 2×300 $= 2 \times 3 \times 100$ $= 6 \times 100$ $= 600$	d. $2 \times 3{,}000$ $= 2 \times 3 \times 1{,}000$ $= 6 \times 1{,}000$ $= 6{,}000$

Powers of 10 Students work with expanded and exponential forms of powers of 10. This enables students to connect factors of 10, 100, and 1,000 to repeated multiplication and exponents.

$10 = 10 \times 1$ exponential form: 10^1

$100 = 10 \times 10$ exponential form: 10^2

$1{,}000 = 10 \times 10 \times 10$ exponential form: 10^3

This concept is extended to multiplying with powers of 10.

$5 \times 10 \times 10 = \underline{5 \times 10^2}$ $3 \times 10^3 = \underline{3 \times 10 \times 10 \times 10}$

Patterns With Fives and Zeros In Lesson 2, students discover that using a pattern of zeros to predict the number of zeros in a product requires special attention for some combinations of factors. For example, the product 2×50 contains one zero in the factors but two zeros in the product.

the product of 2×5 is 10 $2 \times 50 = 100$

Lesson 2 makes students aware that using a pattern of zeros (e.g., counting zeros in the factors) must be done with care when predicting the reasonableness of a product.

Generalizations The real world examples in Lessons 1 and 2 lead students to the following place value generalizations.

- Multiplying by 10 produces a number 10 times as great and shifts each digit one place to the left.
- Multiplying by 100 produces a number 100 times as great and shifts each digit two places to the left.
- Multiplying by 1,000 produces a number 1,000 times as great and shifts each digit three places to the left.

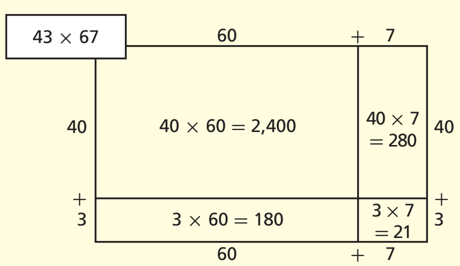
Multidigit Multiplication

Lessons **3** **4** **5**

In these lessons students discuss, analyze, and draw models for contexts involving multiplication of two-digit numbers, and work with methods that can be used to find the products.

Place Value Sections One way to model the multiplication of two-digit numbers, such as 43 × 67, is to use a Place Value Sections model, which shows the relationship the Distributive Property shares with multiplication—the areas of the four smaller rectangles represent the four partial products of the multiplication.

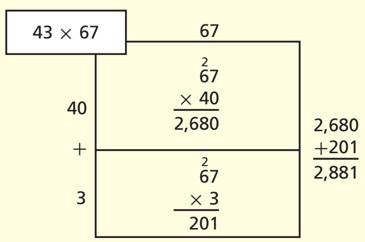
Place Value Sections



Expanded Notation

$$
\begin{array}{r}
67 = 60 + 7 \\
\underline{43 = 40 + 3} \\
40 \times 60 = 2{,}400 \\
40 \times 7 = 280 \\
3 \times 60 = 180 \\
\underline{3 \times 7 = \overset{1}{2}1} \\
2{,}881
\end{array}
$$

Place Value Rows



Short Cut

New Groups Above

$$
\begin{array}{r}
2 \\
2 \\
67 \\
\underline{\times\ 43} \\
201 \\
\underline{2{,}680} \\
2{,}881
\end{array}
$$

New Groups Below

$$
\begin{array}{r}
67 \\
\underline{\times\ 43} \\
{}^{1\,2}81 \\
{}^{2\ \ 2}480 \\
\underline{} \\
\overset{1}{2{,}881}
\end{array}
$$

Distributive Property Using expanded notation to find the product of two-digit numbers involves place value because the Distributive Property decomposes the factors into base ten units, and produces the four factor pairs (40 × 60, 3 × 60, 40 × 7, and (3 × 7) that represent the partial products of the multiplication.

$$
\begin{aligned}
43 \times 67 &= (40 + 3) \times (60 + 7) \\
&= (40 + 3) \times 60 + (40 + 3) \times 7 \\
&= 40 \times 60 + 3 \times 60 + 40 \times 7 + 3 \times 7
\end{aligned}
$$

These lessons demonstrate that although a rectangular area model can be used to represent all multiplication situations, the partial products of those situations can be recorded in different ways.

***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN**

Multi-digit Multiplication Another part of understanding general base-ten methods for multi-digit multiplication is understanding the role played by the Distributive Property. This allows numbers to be decomposed into base-ten units, products of the units to be computed, then combined. By decomposing the factors into like base-ten units and applying the Distributive Property, multiplication computations are reduced to single-digit multiplications and products of numbers with multiples of 10, 100, and 1,000. Students can connect diagrams of areas or arrays to numerical work to develop understanding of general base-ten multiplication methods.

Multiplication With Decimal Numbers

Lessons 6 7 8 9

Decimal Shift Patterns The activities presented in Lesson 6 enable students to compute decimal products symbolically and explore shift patterns in those products. The first pattern they explore shows that the product of a whole number and a decimal has the same number of decimal places as the decimal factor. Since multiplication is repeated addition, addition is used to verify this relationship.

2×0.3 ton $= 0.6$ ton because $0.3 + 0.3 = 0.6$

Students also explore the effect of multiplying by 0.1, 0.01, and 0.001, and again see the pattern that the product of a whole number and a decimal has the same number of decimal places as the decimal factor.

x	0.3	0.03	0.003
2	2×0.3 $= 2 \times 3 \times 0.1$ $= 6 \times 0.1$ $= 0.6$	2×0.03 $= 2 \times 3 \times 0.01$ $= 6 \times 0.01$ $= 0.06$	2×0.003 $= 2 \times 3 \times 0.001$ $= 6 \times 0.001$ $= 0.006$

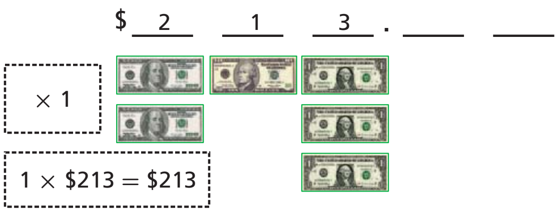
The patterns shown in the table above demonstrate how the product of a whole number and a decimal factor can be computed using ones, tens, and hundreds, or using tenths, hundredths, or thousandths. They give students an opportunity to recognize that the shift pattern for multiplying whole numbers is the opposite of the shift pattern for multiplying decimals. In whole number multiplication, the digits in the multiplicand move to the left. When the multiplication consists of one or more decimal factors, the digits shift to the right. Both shifts often require students to use zeros as placeholders.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Decimal Products and Factors General methods used for computing products of whole numbers extend to products of decimals. Because the expectations for decimals are limited to thousandths and expectations for factors are limited to hundredths at this grade level, students will multiply tenths with tenths and tenths with hundredths, but they need not multiply hundredths with hundredths. Before students consider decimal multiplication more generally, they can study the effect of multiplying by 0.1 and by 0.01 to explain why the product is ten or a hundred times as small as the multiplicand (moves one or two places to the right). They can then extend their reasoning to multipliers that are single-digit multiples of 0.1 and 0.01 (e.g., 0.2 and 0.02, etc.).

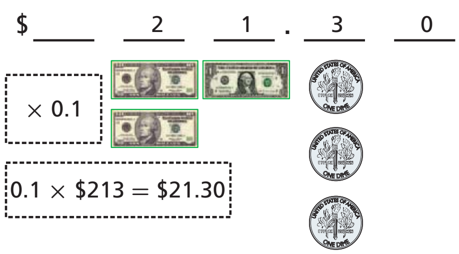
Shift Patterns with Decimals The activities in Lesson 7 provide students with additional opportunities to explore shift patterns of decimals. Students first work with a real world example of saving a portion of a $213 monthly income.

Leon's Earnings



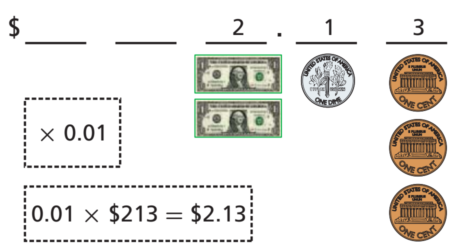
Students first explore the shift using 0.1 as a factor.

Save 0.1 Each Month



Students then explore the shift using 0.01 as a factor.

Save 0.01 Each Month



These activities involve one decimal factor, and lead students to the following generalizations:

- Multiplying by 0.1 produces a shift of one place to the right and produces a product 10 times as small. Multiplying by 0.1 is equivalent to multiplying by $\frac{1}{10}$ or dividing by 10.
- Multiplying by 0.01 produces a shift of two places to the right and produces a product 100 times as small. Multiplying by 0.01 is equivalent to multiplying by $\frac{1}{100}$ or dividing by 100.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Using Reasoning Students can also reason that when they carry out the multiplication without the decimal point, they have multiplied each decimal factor by 10 or 100, so they will need to divide by those numbers in the end to get the correct answer. Also, students can use reasoning about the sizes of numbers to determine the placement of the decimal point. For example, 3.2 × 8.5 should be close to 3 × 9, so 27.2 is a more reasonable product for 3.2 × 8.5 than 2.72 or 272. This estimation-based method is not reliable in all cases, however, especially in cases students will encounter in later grades. For example, it is not easy to decide where to place the decimal point in 0.023 × 0.0045 based on estimation. Students can summarize the results of their reasoning such as those above as specific numerical patterns and then as one general overall pattern such as "the number of decimal places in the product is the sum of the number of decimal places in each factor."

Two Decimal Factors Also in Lesson 7, students explore shift patterns that occur when both factors are decimals. The symbolic exercises and real world problems they work with lead them to infer the following:

- The number of decimal places in a product is the same as the total number of decimal places in the factors.

Connect Decimals to Whole Numbers and Fractions The algorithms used to multiply decimals are the same as those used for whole numbers. Whether the factors are decimals or whole numbers, the digits in the product are the same. Reminding students of this connection will allow them to focus on the relationships between multiplying whole numbers and multiplying decimals and will promote understanding.

The rules about placing a decimal point in the product connects multiplying decimals to fractions. When multiplying fractions, we multiply the numerators as if they are whole numbers, and then multiply the denominators, for example $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$. The number of zeros in the product of the denominator corresponds to the number of decimal places, which we use to place the decimal point.

Application The activities in Lesson 8 give students an opportunity to practice and apply the decimal shift patterns they learned in Lesson 7. Their work involves computing products symbolically and solving problems in real world contexts.

Properties In Lesson 8, students also complete equations using the Commutative Property of Multiplication, the Associative Property of Multiplication, and the Distributive Property.

Compare and Contrast Shift Patterns In Lesson 9, students compare and contrast shift patterns of whole number multipliers (10 and 100) to those of decimal multipliers (0.1 and 0.01).

Whole Number Multipliers	**Decimal Number Multipliers**
1. When you multiply by 10, the number gets __10__ times as big. The digits shift __1__ place(s) to the __left__.	**2.** When you multiply by 0.1, the number gets __$\frac{1}{10}$__ as big. The digits shift __1__ place(s) to the __right__.
3. When you multiply by 100, the number gets __100__ times as big. The digits shift __2__ place(s) to the __left__.	**4.** When you multiply by 0.01, the number gets __$\frac{1}{100}$__ as big. The digits shift __2__ place(s) to the __right__.

Students conclude from these relationships that:

- multiplying by 10 or 0.1 results in a shift of one place.
- multiplying by 100 or 0.01 results in a shift of two places.
- the difference is the direction of the shift.

Powers of 10 Students also explore shift patterns using powers of 10, and infer that the shift pattern of the digits is described by the number of times 10 is used as a factor (i.e. the exponent).

$10^1 = 10$ $10^2 = 10 \times 10 = 100$ $10^3 = 10 \times 10 \times 10 = 1{,}000$

$0.4 \times 10 = 0.4 \times 10^1 = \underline{\ 4\ }$

$0.4 \times 100 = 0.4 \times 10 \times 10 = 0.4 \times 10^2 = \underline{\ 40\ }$

$0.4 \times 1{,}000 = 0.4 \times 10 \times 10 \times 10 = 0.4 \times 10^3 = \underline{\ 400\ }$

Generalization In Lesson 9, students also extend the Big Idea from Lesson 7—the number of decimal places in a product is the same as the total number of decimal places in the factors—to include 5-pattern products, or products such as 0.8×0.5. Although such products are typically written using only one decimal place ($0.8 \times 0.5 = 0.4$), they extend the Big Idea because there are two decimal places in the factors and two in the product ($0.8 \times 0.5 = 0.40$) before the product is written in simplest form.

Reasonable Answers

Lessons 10 11

Check Products The activities and exercises in Lesson 10 remind students that they know a variety of strategies that they can use for checking exact products for reasonableness. One strategy is to use rounding to create an estimate that is compared to an exact answer, and is implemented by students on computations such as these.

	Estimated Answer	Exact Answer
23.	$24 \times 39 \approx \underline{\ 20\ } \times \underline{\ 40\ } \approx \underline{\ 800\ }$	$24 \times 39 = \underline{\ 936\ }$
26.	$12.3 \times 3.7 \approx \underline{\ 12\ } \times \underline{\ 4\ } \approx \underline{\ 48\ }$	$12.3 \times 3.7 = \underline{\ 45.51\ }$

When making estimates, students are encouraged to use patterns and, whenever possible, mental math. For example, a pattern of basic facts and zeros to create an estimate for Exercise 23 above could be $2 \times 4 = 8$ and $2 \times 40 = 80$. So, $20 \times 40 = 800$. The basic fact $2 \times 4 = 8$, and the generalization *the number of zeros in the factors is equal to the number of zeros in the product,* enables many students to complete the estimate using only mental math.

Check for Reasonableness Also in Lesson 10, students use estimation to check the reasonableness of given products, such as:

$24.5 \times 4 = 98$	$0.56 \times 30 = 1.68$	$15.2 \times 2.03 = 30.856$

Although strategies for checking these products may vary, many students will use rounding to decide. For example:

- $24.5 \times 4 = 98$ is reasonable because 24.5 is about 25, and $25 \times 4 = 100$.
- $0.56 \times 30 = 1.68$ is not reasonable because 0.56 is about $\frac{1}{2}$, and $\frac{1}{2}$ of 30 is 15.
- $15.2 \times 2.03 = 30.856$ is reasonable because 15.2 is close to 15, 2.03 is close to 2, and $15 \times 2 = 30$.

Practice The activities in Lesson 11 involve practice, giving students an opportunity to apply the concepts they have learned in Unit 4. The activities range from performing symbolic multiplication computations to solving real world multiplication word problems, and involve either one or two decimal factors.

Focus on Mathematical Practices

Lesson 12

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use what they know about multiplying whole numbers and decimals to complete computations related to measurements of insects.