

Math Expressions
VOCABULARY

As you teach the unit, emphasize understanding of this term.

- prefixes

See the Teacher Glossary.

Getting Ready to Teach Unit 5

Learning Path in the Common Core Standards

In this unit, students explore customary and metric measurement, time concepts, and area and perimeter. An understanding of the base ten structure of the place value system is used to help students understand the metric system of measurement. This system of measurement is an important focus, as it is the system of measurement used in the sciences. It is also the everyday system of measurement used in almost all other countries in the world. Frequent exposure to metric units will help students think about measurement in the metric system.

Students also build on their understanding of measuring distance, weight, capacity, and time using the U.S. customary system, including measuring to the $\frac{1}{8}$ inch. They apply the skills they develop to real-world measurement situations. They also develop their ability to make estimates and to use measurements to solve problems.

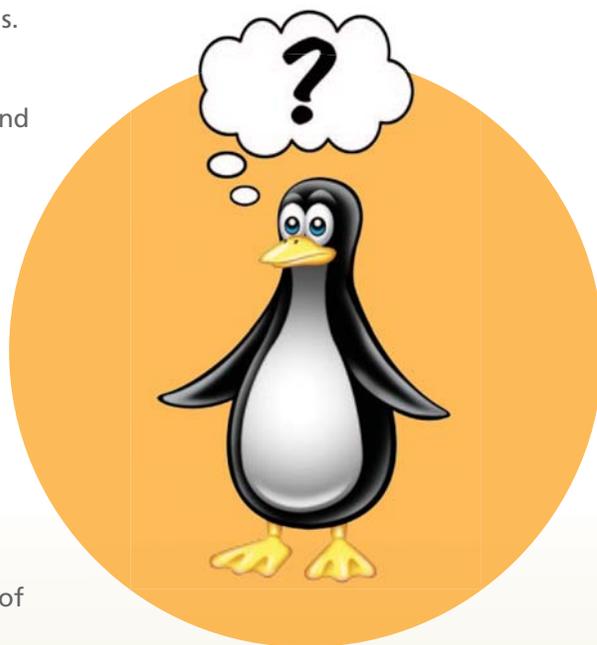
Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit, we use Puzzled Penguin to show typical errors that students make. Students enjoy explaining Puzzled Penguin's error and teaching Puzzled Penguin the correct way to convert measurements and find perimeter and area. The following common errors are presented to the students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin:

- ▶ **Lesson 2:** Choosing the incorrect multiplier when converting liters to milliliters
- ▶ **Lesson 6:** Using the incorrect formula to find the area of a rectangle

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.



Measurement Data

Lessons

1

2

3

4

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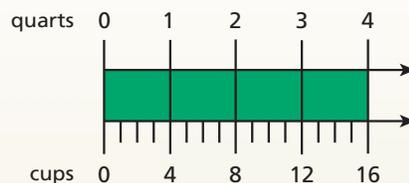
Metric and Customary Systems of Measure In this unit, students describe the attributes of objects, convert from one unit to another, and solve problems within the metric and customary systems of measure. Using units from both systems allows students to develop a deeper understanding of each attribute as well as the relationship between the two systems.

Metric Units Metric units are derived from the base units: meters, liters, and grams. Prefixes are used to name larger and smaller units. Emphasis is placed on the metric units that are most commonly used in everyday life around the world and units with which students may already be familiar. For example, students may know about 2-L bottles or 10-km Olympic races.

Estimation The ability to estimate measurements is an important skill. Often it can replace finding actual measurements or be used to check a measurement or the solution to a measurement problem. Benchmarks are used frequently throughout the unit to help students conceptualize the relative size of different measurements. Students who develop strong personal benchmarks, for example: *my uncle is about 6 feet tall*, *my dog weighs about 9 kilograms*, or *our juice pitcher holds about 12 cups*, are more likely to be able to estimate accurately.

Tables and Double Number Lines Throughout the unit, tables and double number lines are tools used to model unit equivalencies of metric and customary units of measure. Both models help students to convert measurements by facilitating the recognition of patterns in the conversions. The examples below show how students use a table and a double number line to convert customary units of capacity.

Quarts	Fluid Ounces
1	32
2	64
3	96
4	128
5	160
6	192



The use of the double number lines is beneficial because it enables students to use their understanding of the properties of a ruler to help conceptualize measurement conversions. Note that the distance between each quart is divided into 4 equal sections to show that there are 4 cups in a quart. Students can see that since 3 aligns with 12, 3 quarts = 12 cups.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON MEASUREMENT AND DATA

Measurement Data As the name suggests, measurement data comes from taking measurements. For example, if every child in a class measures the length of his or her hand to the nearest centimeter, then a set of measurement data is obtained.

Linear Measurement

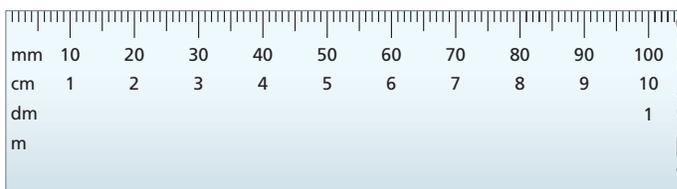
Lessons

1

4

Metric Units of Linear Measurement In Lesson 1, students begin their exploration of measurement by investigating the metric units of linear measure. A ruler is used to help students visualize both the size of metric units and how they are related to each other. A ruler like the one below shows that:

1 decimeter = 10 centimeters = 100 millimeters.



As students explore the properties of a ruler, they are given the opportunity to also investigate the following properties of measurement. *Iteration*: measuring tools have units that repeat. *Partitioning*: measuring tools have large units divided into smaller units that are the same size. *Compensatory Principle*: more smaller units than larger units are needed to measure any distance. *Transitivity*: the relationship among three elements, for example, if Object A is longer than Object B and Object B is longer than Object C, then Object A is longer than Object C.

Students also continue to develop a sense of the relative size of metric units as they identify the appropriate unit to measure common lengths. Common benchmarks can help students develop a sense of the sizes of metric units. For example, things that are typically measured in feet or yards are measured in meters. Greater lengths, such as distances between cities, are measured in kilometers. Shorter lengths, such as the length of a pencil, are measured in centimeters, and very small lengths are measured in millimeters.

To help students understand the relationship among the linear units of metric measure, a table like the one below is presented.

Units of Length						
kilometer	hectometer	decameter	meter	decimeter	centimeter	millimeter
km	hm	dam	m	dm	cm	mm
$10 \times 10 \times 10 \times$ larger	$10 \times 10 \times$ larger	$10 \times$ larger	1 m	$10 \times$ smaller	$10 \times 10 \times$ smaller	$10 \times 10 \times 10 \times$ smaller
1 km = 1,000 m	1 hm = 100 m	1 dam = 10 m		10 dm = 1 m	100 cm = 1 m	1,000 mm = 1 m

Students are able to apply their understanding of place value and powers of ten to convert from one measurement to another. The meter is the base unit. The larger units of measure are to the left of the meter and the smaller units are to the right. As students move from left to right on the table, they multiply. As they move from right to left, they divide. Students learn that the prefixes of each measurement indicate its magnitude.

Customary Units of Linear Measurement In Lesson 4, students continue their work with linear measurement as they use a ruler to measure and convert among customary units of measure. They extend their measurement skills as they identify measurements through the nearest $\frac{1}{8}$ inch. For example, students identify the length of the line segment below as being at $3\frac{7}{8}$ inches.



They realize that measuring to the nearest $\frac{1}{8}$ inch is more precise than measuring to the nearest $\frac{1}{2}$ or $\frac{1}{4}$ inch.

Students use standard equivalencies to convert among inches, feet, yards, and miles. They use tables and double number lines as tools to help them.

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Fractional Lengths [Students] use their developing knowledge of fractions and number lines to extend their work from the previous grade by working with measurement data involving fractional measurement values.

Capacity, Mass, and Weight

Lessons

2
5

Metric Units of Liquid Capacity Capacity is a measure of volume usually associated with liquid measurement. Students use benchmarks to help them determine the size of metric units of capacity. For example, one liter is a little more than one quart. Bottled water, juices, and carbonated drinks often come in one-liter or two-liter containers. Smaller quantities of liquids are typically labeled in milliliters. Milliliters and grams are used in this country for prescriptions. As with metric units of length, a table is presented to show the relationship among metric units of liquid capacity. The liter is the base unit.

Units of Liquid Volume						
kiloliter	hectoliter	decaliter	liter	deciliter	centiliter	milliliter
kL	hL	daL	L	dL	cL	mL
$10 \times 10 \times 10 \times$ larger	$10 \times 10 \times$ larger	$10 \times$ larger	1 L	$10 \times$ smaller	$10 \times 10 \times$ smaller	$10 \times 10 \times 10 \times$ smaller
1 kL = 1,000 L	1 hL = 100 L	1 daL = 10 L		10 dL = 1 L	100 cL = 1 L	1,000 mL = 1 L

Customary Units of Capacity In the customary system, as students see in Lesson 5, the volume of liquids is measured in fluid ounces, cups, quarts, and gallons. One cup of milk, for example, will fit into a container with a capacity of one cup. Teaspoon and tablespoon conversions are also presented. Students use standard conversions among the various measurement units, as well as use their estimation skills to determine the appropriate unit to measure common containers.

Weight and Mass Weight is dependent on the effects of gravity, but mass is a measurement independent of gravity. A person who weighs 150 pounds on Earth weighs less on the moon, but still has the same mass. Because we all live on Earth, we sometimes talk in everyday terms about something “weighing 100 grams.”

Metric Units of Mass The gram is the basic unit of mass. One gram is a very small unit. A paper clip or peanut has a mass of about one gram. Students use a table like the ones for meters and liters to convert among the metric units of mass.

Units of Mass						
kilogram	hectogram	decagram	gram	decigram	centigram	milligram
kg	hg	dag	g	dg	cg	mg
$10 \times 10 \times 10 \times$ larger	$10 \times 10 \times$ larger	$10 \times$ larger	1 g	$10 \times$ smaller	$10 \times 10 \times$ smaller	$10 \times 10 \times 10 \times$ smaller
1 kg = 1,000 g	1 hg = 100 g	1 dag = 10 g		10 dg = 1 g	100 cg = 1 g	1,000 mg = 1 g

Customary Units of Weight In Lesson 5, students explore the customary units of weight, ounces, pounds, and tons. The ton is a new unit for students. To help them understand the magnitude of the customary units of weight, students find benchmarks for each measurement unit. They use standard equivalencies to convert among the units.

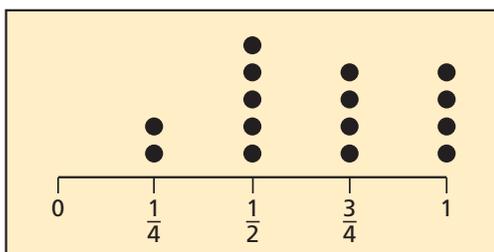
Lesson

3

Time

Time Conversions In Lesson 3, students use time unit equivalencies to convert among time units. The units students explore include seconds, minutes, hours, days, weeks, months, and years.

Time, Fractions, and Line Plots Working with time provides opportunities to review what students have learned about fractions. It is common to relate fractions of an hour to minutes in order to determine that $\frac{1}{4}$ hour is 15 minutes and $\frac{3}{4}$ hour is 45 minutes. Students apply this understanding as they interpret line plots that include fractions of an hour.



Time Spent Reading Each Night (in hours)

Students use the line plot to determine, for example, that the greatest number of students spent $\frac{1}{2}$ hour a night reading.

Elapsed Time In the previous grade, students found elapsed time in hours and minutes and used these skills to solve real world problems. Students review these skills in Lesson 3. The principle that clocks are comprised of iterated units, like all measuring tools, is reinforced as students count the sectors through which the clock hands have traveled to find elapsed time.

Using subtraction to find elapsed time is also presented. For example, to find the time that a student practiced his instrument if he started at 8:21 A.M. and ended at 9:35 A.M. can be found this way:

$$\begin{array}{r} 9:35 \text{ A.M.} \\ - 8:21 \text{ A.M.} \\ \hline 1 \text{ hr } 14 \text{ min} \end{array}$$

Finding elapsed time across noon and midnight is also addressed. Students realize that they have to find the elapsed time between the starting time and either noon or midnight, then find the elapsed time from noon or midnight to the end time. These times, added together, give the total elapsed time.

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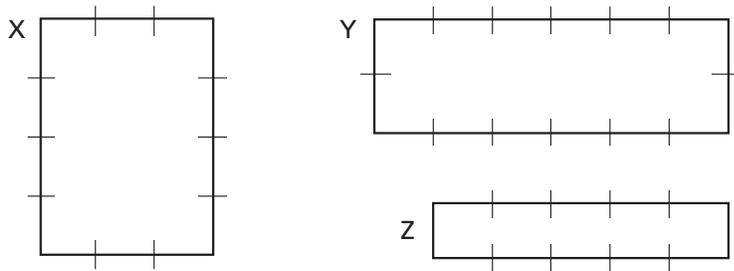
Line Plots The Standards call for students to represent measurement data with a *line plot*. This is a type of display that positions the data along the appropriate scale, drawn as a number line diagram. These plots have two names in common use, “dot plot” (because each observation is represented as a dot) and “line plot” (because each observation is represented above a number line diagram). The number line diagram in a line plot corresponds to the scale on the measurement tool used to generate the data. In a context involving measurement of liquid volumes, the scale on a line plot could correspond to the scale etched on a graduated cylinder.

Perimeter and Area

Lesson

6

Perimeter The presentation of perimeter in Lesson 6 begins with emphasizing a conceptual understanding of perimeter. Students learn that perimeter is the distance around a figure and it is measured in linear units. The following models help students conceptualize the linear units in the perimeter of figures.



Key:

 = 1 cm

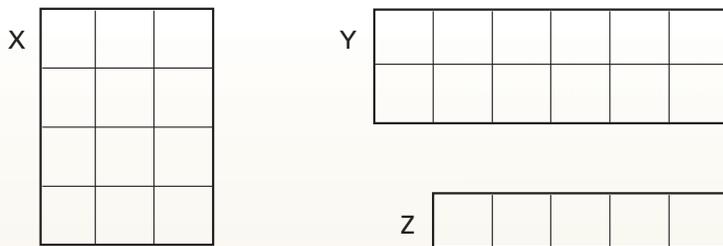
Length = l

Width = w

Perimeter = P

Through a variety of experiences in which students find the perimeter of rectangles, they are able to generalize the formula: $P = l + w + l + w$. Students' basic knowledge about rectangles lays the conceptual foundation for this simple formula development. Students are encouraged to discuss how multiplication can be used to write this formula in a shorter way; for example: $P = (l \cdot 2) + (w \cdot 2)$ or $P = (l + w) \cdot 2$.

Area As students begin to explore area, they again concentrate on conceptualizing what area is and what units are used to measure area. Students learn that area is the number of same-sized units that cover a shape with no gaps or overlaps. Area is measured in square units. The following models help students visualize area.



Key:

 = 1 sq cm

Length = l

Width = w

Area = A

The models above facilitate students understanding that they can find the area of the rectangle by calculating the number of squares in an array. Students call upon their prior understanding of the area model as pushed-together squares and that the number of squares in an array can be found by multiplying the number of squares in each row by the number of rows. They use this knowledge to help them develop and understand the area formula: $A = l \cdot w$.

Problem Solving

Lessons

3

5

7

Problem Solving Plan In *Math Expressions*, a research-based problem-solving approach that focuses on problem types is used.

- Interpret the problem.
- Represent the situation.
- Solve the problem.
- Check that the answer makes sense.

Time Problems Students apply their understanding of time concepts to solve problems involving elapsed time. A variety of problems are presented in which students need to find the start time, the end time, or the time that has elapsed.

Measurement, Perimeter, and Area Problems Students apply their understanding of measurement and operations as they solve problems in which they have to add, subtract, multiply, or divide measurements. They also solve problems involving perimeter and area. To successfully solve the problems presented in this unit, students need to understand the structure of the problem to identify the operation needed, as well as be able to use conversion equivalences to flexibly move from one unit to another.

Focus on Mathematical Practices

Lesson

8

The standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use what they know about measurement, area, and perimeter to solve problems involving gardening.