Getting Ready to Teach Unit 1

Learning Path in the Common Core Standards

In previous grades, students have developed an understanding of place value and addition and subtraction of numbers through one thousand. The uniformity of the base-ten system allows students to generalize their understanding of place value and computation of lesser numbers to greater numbers. Unit 1 broadens and deepens students' experiences with numbers through one million.

Students have learned that the concepts of grouping and ungrouping numbers are a key step in performing multidigit addition and subtraction. The activities in this unit help students gain practical understanding of addition and subtraction and the relationship between the two operations. Estimation and mental math provide students with methods to validate their answers.

Math Expressions VOCABULARY

As you teach the unit, emphasize understanding of these terms.

- break-apart diagram
- ungroup
- place value drawings

See the Teacher Glossary

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit, we use Puzzled Penguin to show typical errors that students make. Students enjoy explaining Puzzled Penguin's error and teaching Puzzled Penguin the correct way to add and subtract or use place value. The following common errors are presented to the students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin.

- **Lesson 4:** Incorrectly translating zeros in the standard form of a number to the word form
- **Lesson 8:** Forgetting to make a new group of hundreds when addition results in 10 tens
- **Lesson 10:** When ungrouping hundreds to subtract, forgetting to reduce the number of hundreds by 1
- **Lesson 13:** Not writing the correct addition equation to model and solve a real world problem

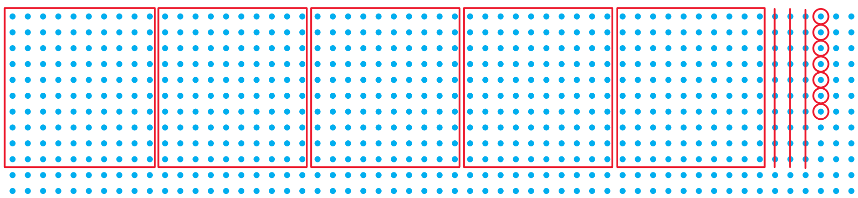
In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

Place Value Concepts

Lessons 1 2 4

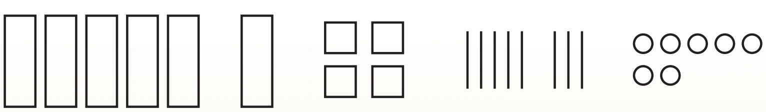
Place Value Drawings In *Math Expressions*, students use place value drawings to help them conceptualize numbers and understand the relative sizes of place values. In the early lessons of this unit, students make these drawings on the dot-array side of their MathBoards. The drawing below represents the number 537. It shows:

- 5 hundred boxes (5 squares that each contain 100 dots) = 500.
- 3 quick tens (5 line segments that each connect 10 dots) = 30.
- 7 ones (7 circles that each contain 1 dot) = 7.



The place value drawing is a beneficial model because it helps students visualize the magnitude of numbers. For example, in this model, students can see that the 5 in the hundreds place represents 500 dots and they can develop a sense of the size of 500 dots.

Once students have a conceptual understanding of the number of 1s contained inside each place, they move to drawings without dots. For example, the drawing below shows 6,487. Because these types of drawings do not need to be perfectly scaled, students can make them quickly. (Grouping the shapes by fives, as in this drawing, makes them easier to count.)

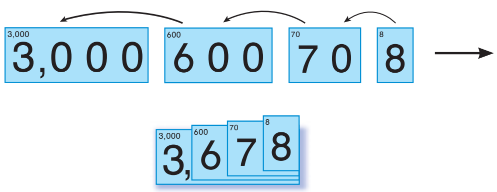


6 thousands 4 hundreds 8 tens 7 ones

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Base-ten Units The power of the base-ten system is in repeated bundling by ten: 10 tens make a unit called a hundred. Repeating this process of creating new units by bundling in groups of ten creates units called thousand, ten thousand, hundred thousand . . . .

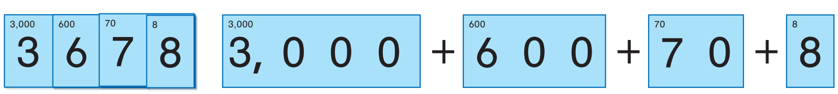
Whole Number Secret Code Cards Students explore place value by assembling Secret Code Cards to form multidigit numbers. The cards show place values. To make the number 3,678, students select the cards representing 3 thousands, 6 hundreds, 7 tens, and 8 ones.



Using the cards is beneficial for students because the cards emphasize how the position of the digit in the number determines the value of the digit. For example, with the cards students can more easily see that 3 on the thousands card is 3,000 while a 3 on the tens card is 30.

The uniform use of the cards to show the standard, expanded, and word form of numbers allows students to make connections among the different forms.

Standard Form **Expanded Form**

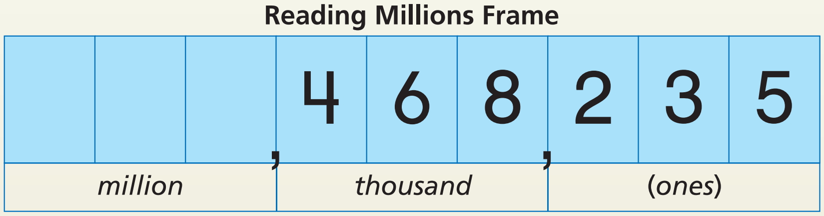


Word Form The backs of the cards show the values in word form.

three thousand	six hundred	seventy (seven tens)	eight

Read and Write Greater Numbers In Lesson 4, students extend their understanding of place value to include numbers through one million. The place value patterns observed in Lesson 2 provide students with the foundation on which they can build an understanding of greater numbers. Using the Secret Code Cards and organizing the cards in the Reading and Writing Millions Frame facilitates students' ability to represent, build, read, and write greater numbers.

Reading Millions Frame



from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

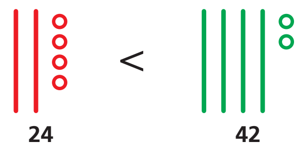
Position The base-ten system is a remarkably efficient and uniform system for systematically representing all numbers. Using only the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, every number can be represented as a string of digits, where each digit represents a value that depends on its place in the string.

Rounding and Comparing Numbers

Lessons 3 5

Comparing The place value work in this unit extends students' understanding to numbers through one million. An understanding that in the base-ten system one of a greater unit is always greater than nine or fewer of a lesser unit provides students with the foundation necessary to compare numbers of increasing value.

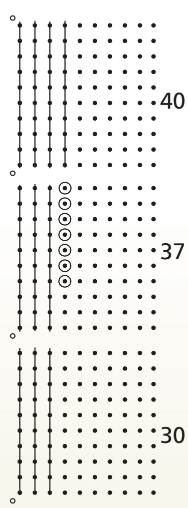
To help students connect their understanding of place value to comparing, they are encouraged to make place value drawings. The place value drawing below emphasizes that the position of the digits is important and affects the magnitude of the number.



As students' understanding of place value deepens in Grade 4, the ability to compare numbers also expands. For example, the knowledge that 3 ten thousands is greater than 8 thousands allows them to determine that:

$$32{,}119 > 8{,}932$$

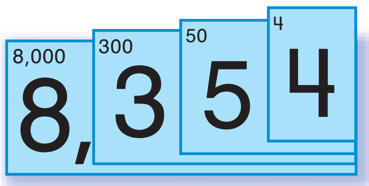
Rounding In this unit, students build upon the understanding of rounding that they acquired in Grade 3. Students use dot array drawings to represent numbers they are rounding. Dot array drawings are useful when rounding lesser numbers because they help students to more easily visualize to which unit a number is closer. For example, the model below shows that 37 is closer to 40 than to 30.



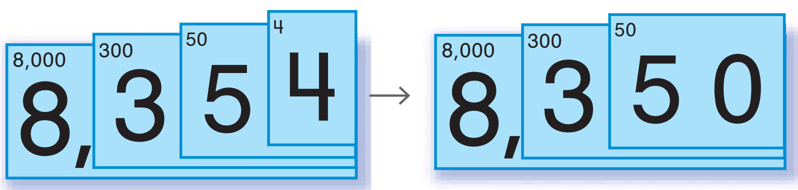
from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Comparing Comparing magnitudes of two-digit numbers draws on the understanding that 1 ten is greater than any amount of ones represented by a one-digit number. Comparing magnitudes of three-digit numbers draws on the understanding that 1 hundred (the smallest three-digit number) is greater than any amount of tens and ones represented by a two-digit number.

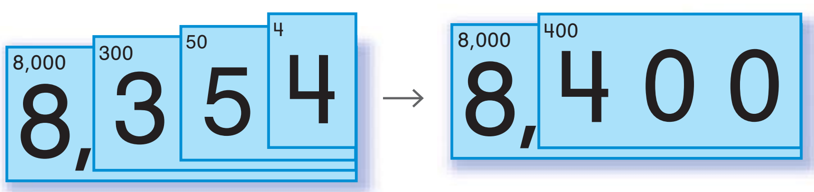
Secret Code Cards As students extend their rounding experiences to include thousands, they use Secret Code Cards to represent numbers. To round 8,354, they use the cards to make the number. Then they identify the place to which they need to round.



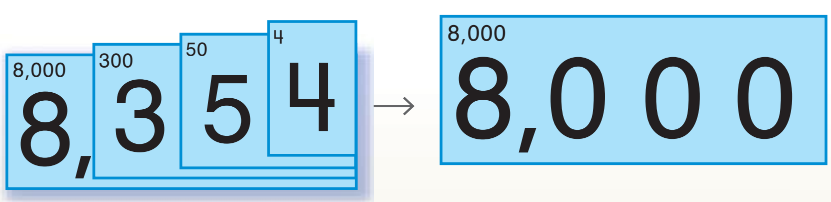
Round to the Nearest Ten They need to decide if they will keep the 50 card or change it to a 60 card.



Round to the Nearest Hundred They need to decide if they will keep the 300 card, or trade it in for the 400 card.



Round to the Nearest Thousand They need to decide if they should keep the 8,000 card, or trade it in for the 9,000 card.



Round Greater Numbers Using the cards facilitates students' ability to identify the place to which they are rounding and helps them focus their attention on which place they need to use to determine how to round. As with the skill of comparing, the focus on using place value to round allows students to successfully use what they know about rounding lesser numbers and the base-ten system to round greater numbers. For example, students learn to round greater numbers to the thousands place.

638,734 ⟶ 639,000

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Rounding Students use their place value understanding to round numbers to the nearest 10 or 100. They need to understand that when moving to the right across the places in a number (e.g., 456), the digits represent smaller units. When rounding to the nearest 10 or 100, the goal is to approximate the number by the closest number with no ones or no tens and ones (e.g., so 456 to the nearest ten is 460; and to the nearest hundred is 500).

Addition and Subtraction of Whole Numbers

Lessons

6 7 9 10 11 12

Computations The uniformity of the base-ten system facilitates understanding and the extension of place value concepts, but it also provides the foundation for successfully completing standard algorithms for computation within the base-ten system. Once students understand that numbers are composed of ones, tens, hundreds, and so on, they can use this understanding to decompose and compose units in computations.

Once students gain proficiency in adding and subtracting numbers in the hundreds and thousands, they can apply their understanding to add and subtract greater numbers. The algorithms taught involve combining base-ten units and therefore can be generalized to computing with greater numbers.

Several different algorithms are presented in this unit. The variety of algorithms is beneficial for students because it allows them to choose the algorithm that best suits their learning style and the one that feels the most natural to them. The algorithms themselves emphasize grouping and ungrouping to address common errors that are learning obstacles for students.

Numeric Addition Methods Lessons 6 and 7 present four algorithms for adding numbers with addends through one million. The examples below show how to use these four numeric methods to add 3,246 and 2,197. Students learn that the order in which the steps of the method are recorded does not change the value of the answer.

New Groups Above

$$\begin{array}{r} {\scriptstyle 11}\;\; \\ 3{,}246 \\ +\ 2{,}197 \\ \hline 5{,}443 \end{array}$$

New Groups Below

$$\begin{array}{r} 3{,}246 \\ +\ 2{,}197 \\ \hline {\scriptstyle 11}\; \\ 5{,}443 \end{array}$$

Subtotal Left to Right

$$\begin{array}{r} 3{,}246 \\ +\ 2{,}197 \\ \hline 5{,}000 \\ 300 \\ 130 \\ +\ 13 \\ \hline 5{,}443 \end{array}$$

Subtotal Right to Left

$$\begin{array}{r} 3{,}246 \\ +\ 2{,}197 \\ \hline 13 \\ 130 \\ 300 \\ +\ 5{,}000 \\ \hline 5{,}443 \end{array}$$

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Computations Standard algorithms for base-ten computations with the four operations rely on decomposing numbers written in base-ten notation into base-ten units. The properties of operations then allow any multidigit computation to be reduced to a collection of single-digit computations. These single-digit computations sometimes require the composition or decomposition of a base-ten unit.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

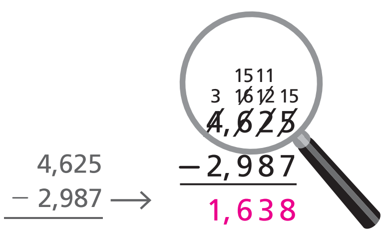
Algorithms In mathematics, an algorithm is defined by its steps and not by the way those steps are recorded in writing. With this in mind, minor variations in methods of recording standard algorithms are acceptable.

Numeric Subtraction Methods Lessons 9 and 11 present algorithms for subtracting numbers up to one million. The subtraction algorithms parallel the addition algorithms students have learned. This parallel presentation benefits students because they can use their understanding of place value and addition to subtract. As with addition, various methods are taught so that students can choose the method that aligns most closely to their learning style.

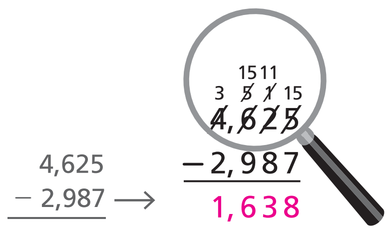
In numeric addition methods, students use composition to group units of lesser values to units of greater values, for example, grouping 10 ones as 1 ten, or 10 thousands as 1 ten thousand. With subtraction, students use the opposite action. They decompose numbers to ungroup units of greater value into units of lesser value. For example, they ungroup 1 hundred thousand to 10 ten thousands or 1 thousand to 10 hundreds.

To address students' common errors in problems where it is necessary to ungroup multiple times, students are encouraged to do all the ungrouping before subtracting. Students can ungroup either from left to right or from right to left. Once everything is ungrouped, students can subtract the places in any order.

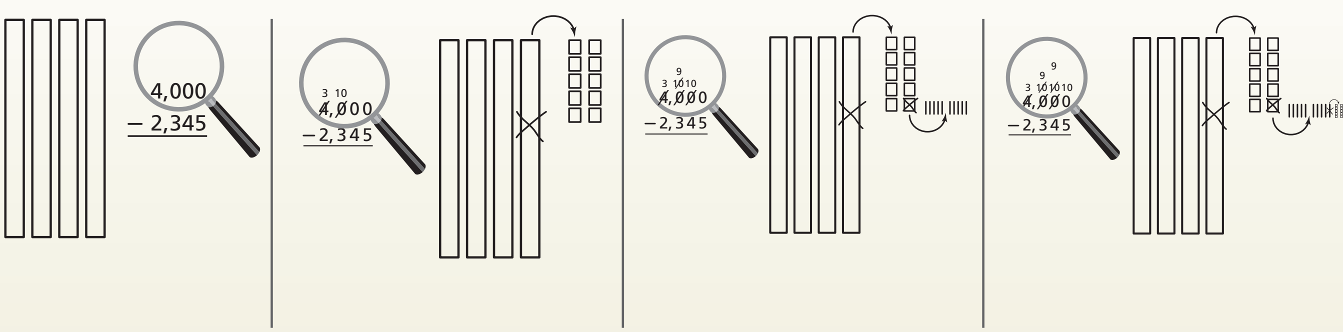
Ungroup from Left to Right



Ungroup from Right to Left



Subtracting Across Zeros In Lesson 9, students begin by subtracting across zeros for minuends in the thousands. By presenting the place value blocks along with the numeric method, students connect the conceptual underpinnings of subtraction with each algorithmic step.



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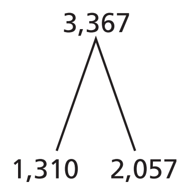
Composing and Decomposing

These algorithms rely on adding or subtracting like base-ten units (ones with ones, tens with tens, hundreds with hundreds, and so on) and composing or decomposing base-ten units as needed (such as composing 10 ones to make 1 ten or decomposing 1 hundred to make 10 tens).

Inverse Operations Lesson 10 presents addition and subtraction as inverse operations. A break-apart diagram is used to show students how the addends and sum are related to the numbers in a subtraction problem.



In problems that describe two parts being put together to get a total, the top number in the diagram is missing. In problems that describe the total being taken apart, one of the bottom numbers is missing. By thinking about addition and subtraction in terms of a total and two parts, students can understand the relationship between addition and subtraction as well as use the break-apart diagram to represent the relationships in word problems.

Students also apply the inverse relationship between addition and subtraction by using addition to check the answers to subtraction problems.

Fluency In grade 4, students are expected to develop fluency in adding and subtracting whole numbers. This fluency is accomplished by students making the connection that, regardless of the number of digits in the numbers they are adding or subtracting, the same process can be used to complete the computation. Lesson 12 gives students the opportunity to continue to develop this essential fluency as well as apply these skills to solve problems.

Estimation and Mental Math

Lesson **8**

Reasonable Answers Many common errors in computation can be caught through using estimation to verify the reasonableness of an answer. Whether students are using a calculator or using algorithms to compute with paper and pencil, estimation gives students a ball-park answer. When students compare their answer to the estimate, they can determine if their answer is reasonable. For example, in the problem 833 + 398, the estimate is 1,200. If students forget to regroup the 10 hundreds as 1 thousand, their answer will not be close to the estimate.

Mental Math By presenting students with opportunities to adjust their estimates and look for easy combinations of numbers to add mentally, students develop flexibility with numbers. Developing flexibility will allow students to become more efficient and fluent with computation. Additionally, in this unit, as students decide whether a problem needs an exact answer or if an estimate is sufficient and then determine whether they can use a special strategy or a generalized algorithm to find an exact answer, they are developing proficiency in using appropriate tools strategically.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Fluency Use of the standard algorithms can be viewed as the culmination of a long progression of reasoning about quantities, the base-ten system, and the properties of operations.

Problem Solving

Lessons **12** **13**

Problem Solving Plan In *Math Expressions* a research-based problem-solving approach that focuses on problem types is used.

- Interpret the problem
- Represent the situation
- Solve the problem
- Check that the answer makes sense

Multistep Problems The equations that students write for problems with more than one step may include more than one operation. Sometimes the order in which these operations are completed is not important. In other problems, students must find the answer to a hidden question and use that answer to answer the question of the problem. We use the term hidden questions to make the conceptual point that students may need to answer these questions even if they do not appear in the problem.

Writing Equations When students read and represent problems, they often translate the words in the order they appear in a problem. This literal translation may result in an equation that represents the problem but is not in a form that students can use to find the solution. This unit emphasizes using a break-apart diagram or bar diagram to represent addition and subtraction problems. Students use the models to write a situation equation and a solution equation to solve word problems.

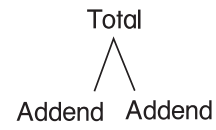
Focus on Mathematical Practices

Lesson **14**

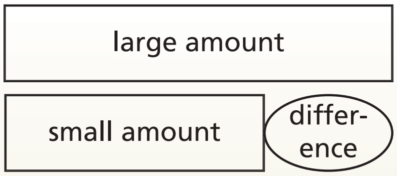
The standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use what they know about adding and subtracting whole numbers to add, subtract, and compare the lengths of bridges.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Word Problems Fourth graders extend problem solving to multistep word problems using the four operations posed with whole numbers. The same limitations discussed for two-step problems concerning representing such problems using equations apply here. Some problems might easily be represented with a single equation, and others will be more sensibly represented by more than one equation or a diagram and one or more equations.



Break-Apart Diagram



Comparison Bar Diagram