Getting Ready to Teach Unit 5

Learning Path in the Common Core Standards

In this unit, students study problem solving and problem types. They use their knowledge of carrying out operations, inverse operations, reading problems, and properties of operations to develop proficiency in solving one step and two step problems.

Visual models and real world situations are used throughout the unit to illustrate important problem solving concepts.

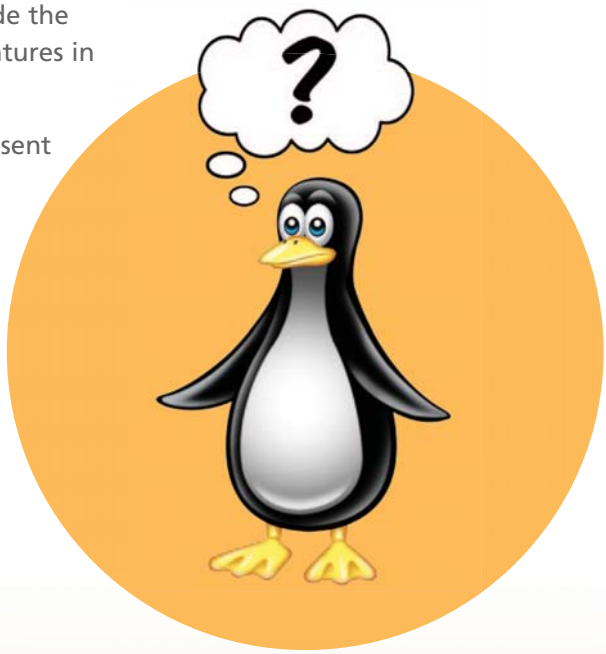
Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit, we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, why this way is correct, and why Puzzled Penguin made the error. Common errors are presented in Puzzled Penguin features in the following lessons:

- **Lesson 5:** drawing comparison bars incorrectly to represent a problem situation
- **Lesson 8:** forgetting to do the second step in a two step problem

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.



***Math Expressions* VOCABULARY**

As you teach this unit, emphasize understanding of these terms.

- situation equation
- solution equation
- comparison situation
- comparison bars

See the Teacher Glossary.

The Problem Solving Process

Using the Mathematical Practices Throughout the program, *Math Expressions* integrates a research-based algebraic problem-solving approach that focuses on problem types. Problem solving is a complex process that involves all eight of the CCSS Mathematical Processes. It is also an individual process that can vary considerably across students. Students may conceptualize, represent, and explain a given problem in different ways.

Mathematical Process	Student Actions
Understand the Problem Situation MP.1 Make sense of the problem. MP.2 Reason abstractly and quantitatively.	**Make Sense of the Language** Students use the problem language to conceptualize the real world situation.
Represent the Problem Situation MP.4 Model with mathematics. MP.7 Look for and make use of structure.	**Mathematize the Situation** Students focus on the mathematical aspects of the situation and make a math drawing and/or write a situation equation to represent the relationship of the numbers in the problem.
Solve the Problem MP.5 Use appropriate tools. MP.8 Use repeated reasoning.	**Find the Answer** Students use the math drawing and/or the situation/solution equation to find the unknown.
Check That the Answer Makes Sense MP.3 Critique the reasoning of others. MP.6 Attend to precision.	**Check the Answer in the Context of the Problem** Students write the answer to the problem including a label. They explain and compare solutions with classmates.

Students are taught to make their own math drawings. Relating math drawings to equations helps them understand where the total and the product is for each operation, and helps them solve equations with difficult unknowns.

Math Talk Learning Community

Research In the NSF research project that led to the development of *Math Expressions*, much work was done with helping teachers and students build learning communities within their classrooms. An important aspect of doing this is *Math Talk*. The researchers found three levels of *Math Talk* that go beyond the usual classroom routine of students simply solving problems and giving answers and the teacher asking questions and offering explanations. It is expected that at Grade 3, students will engage in talk at all levels.

Math Talk Level 1 A student briefly explains his or her thinking to others. The teacher helps students listen to and help others, models fuller explaining and questioning by others, and briefly probes and extends students' ideas.

Example Word Problem

Meg bought 3 mystery books and put them on the shelf with her other mystery books. How many mystery books are on the shelf now?

Is there a way to solve this problem?

Jonelle: I don't think so. We need to know how many mystery books Meg had on her shelf.

What information could we add to the problem to solve it?

Christopher: We could say that Meg bought 3 mystery books and put them on the shelf with her 11 other mystery books.

Math Talk Level 2 A student gives a fuller explanation and answers questions from other students. The teacher helps students listen to and ask good questions, models full explaining and questioning (especially for new topics), and probes more deeply to help students compare and contrast methods.

Example Word Problem

Eight vans with the same number of students in each van took 40 students to the science center for a field trip. How many were in each van?

What do we need to find in the above word problem?

Elijah: the number of students in each van

Brianna: We can write the solution equation, $8 \times s = 40$ to find the answer. s represents the unknown factor for the number of students.

Does s represent the unknown number of groups or the unknown group size?

Emily: the unknown group size

Devon: I wrote $40 \div 8 = s$ to solve this problem.

Who can explain why this equation will work?

Matt: Division and multiplication are inverse operations. One undoes the other.

Math Talk Level 3 The explaining student manages the questioning and justifying. Students assist each other in understanding and correcting errors and in explaining more fully. The teacher monitors and assists and extends only as needed.

Example Word Problem

Katie had 8 dimes and some nickels in her duck bank. She had 4 more nickels than dimes. She took out 5 nickels to buy a newspaper. How many nickels are in her duck bank now?

Will someone share how they solved this problem?

Carlo: Well, I started by figuring out that eight dimes is the same as 80¢. But that's not what the problem asked. So I added 4 to 8 to find out how many nickels Katie had. She had 12 nickels.

Anne: Why did you add 4 to 8?

Carlo: Since Katie has 4 more nickels than dimes, you need to add 4 to the number of dimes, 8.

Kassler: This is a two step problem and I found the same answer as Carlo but then I subtracted 5 from 12. Now Katie has 7 nickels.

Lewis: I got 8 nickels. I wrote $8 + 4 = 12$ and $12 - 5 = 8$ to solve the problem.

Shaina: Your equations are correct, but you subtracted wrong. $8 + 5$ is 13 so $12 - 5 = 8$ is not correct.

Bob: You could also write one equation, $8 + 4 - 5 = 7$ to solve the problem.

Why is it a good idea to read through a problem before you try to solve it?

Lewis: That way you know what the problem is about and can decide what you need to do first.

Summary Math Talk is important not only for discussing solutions to word problems but also for any kind of mathematical thinking students do, such as explaining why a number is even or odd or how to use a drawing to subtract or how to find the area of a rectilinear figure.

Problems with Addition, Subtraction, or Multiplication

Lessons

1 2 3 4 5

Situation and Solution Equations To solve word problems in the lessons in this unit, students may need to write a situation equation and a solution equation.

A *situation equation* shows the structure or relationship of the information in a problem. After writing a situation equation to represent a problem, students use their understanding of inverse operations to rewrite the equation as a solution equation.

A *solution equation* shows what operation is needed to solve a problem. Sometimes the situation equation is also the solution equation.

Addition and Subtraction Problems Students work within addition and subtraction contexts, and represent these problems with Math Mountains, which are also called break-apart drawings. In a Math Mountain for addition and subtraction, the total is placed at the top. A Math Mountain models four related addition and subtraction equations.

$4 + 6 = 10$ $\quad$ $10 - 6 = 4$

$6 + 4 = 10$ $\quad$ $10 - 4 = 6$

Problem Types for Addition and Subtraction Addition and subtraction problems can be classified by the action in the situation and the unknown part. Encourage students to learn the problem types and to use what they know about equations and inverse operations to find a way to solve problems. The examples below are from lessons in this unit.

Put Together: Unknown Addend

Stacy invited 90 girls and some boys to her party. 160 children were invited in all. How many boys were invited?

***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN**

Use place value understanding and properties of operations to perform multidigit arithmetic Students continue adding and subtracting within 1000. They achieve fluency with strategies and algorithms that are based on place value, properties of operations, and/or the relationship between addition and subtraction. Such fluency can serve as preparation for learning standard algorithms in Grade 3, if the computational methods used can be connected with those algorithms.

Take Apart: Unknown Addend

There were 150 people at the park. 70 were playing soccer. The others were playing softball. How many people were playing softball?

Add To: Unknown Addend

Jan planted 80 tulips last week. Today she planted some lilies. Now she has 170 flowers. How many lilies did she plant?

Take From: Unknown Addend

Tim's team had 140 tennis balls. Then his brother's team borrowed some. Now Tim's team has 60 tennis balls. How many did his brother's team borrow?

Add To: Unknown Start

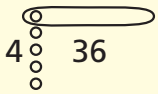
Greta puts some beads on a string. Then she puts on 70 more beads. Now there are 130 beads on the string. How many beads did she put on the string to start?

Take From: Unknown Start

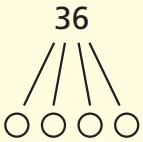
Greta puts some beads on a string. Seventy of the beads fell off the string. Sixty beads are still on the string. How many beads were there at first?

Multiplication Problems with Unknown Factors Students can use drawings to help them write equations to find unknown factors. They should be familiar with both Fast Array drawings and Equal Shares drawings.

Fast Array Drawing



Equal Shares



Students can use either kind of drawing to represent and solve **Unknown Factor** problems like the one below.

Daniel is setting up seats for the third grade play. There are 6 seats in each row. There are 54 seats in all. How many rows of seats are there?

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Grade 3 Students focus on understanding the meaning and properties of multiplication and division and on finding products of single-digit multiplying and related quotients. These skills and understandings are crucial; students will rely on them for years to come as they learn to multiply and divide with multidigit whole numbers and to add, subtract, multiply, and divide with fractions and with decimals.

Additive Comparison Problems Students may correctly represent and explain a given problem in different ways. This is especially true for additive comparison problems, where students easily reverse the relationships especially for misleading language problems. Making a drawing helps students see which amount is larger so they can decide on a correct solution equation. As part of their preparation for comparison problems, students review comparing whole numbers.

Questions about comparison situations can be asked in two ways. Students can make a comparison drawing or use comparison bars to represent the situation. Emphasize that comparison bars do not need to represent the exact difference between the numbers because they are a math tool that shows the larger number, the smaller number, and the difference.

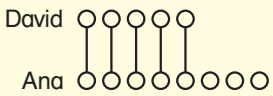
David has 5 marbles. Ana has 8 marbles.

15. How many more marbles does Ana have than David? 3 marbles

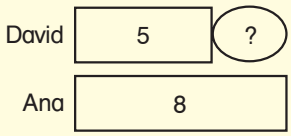
16. How many fewer marbles does David have than Ana? 3 marbles

Here are two ways to represent the comparison situation.

Comparison Drawing



Comparison Bars



Comparison problems that third-grade students solve will have either the larger or smaller amount as the unknown.

Unknown Larger Amount

Maribel has 18 stickers. Arnon has 13 more stickers than Maribel. How many stickers does Arnon have?

Unknown Smaller Amount

Arnon has 31 stickers. Maribel has 13 fewer stickers than Arnon. How many stickers does Maribel have?

***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING**

Using a letter for the unknown quantity, the order of operations, and two step word problems with all four operations Students in Grade 3 begin the step to formal algebraic language by using a letter for the unknown quantity in expressions or equations for one and two step problems. But the symbols of arithmetic, × or • for multiplication and ÷ or / for division, continue to be used in Grades 3, 4, and 5.

Language of Comparison Problems Students work with the language of comparison word problems in Lesson 5, and are presented with *misleading* language and with comparison problems that do not use the terms *more* and *fewer*. For example, the word *more* in the problem below suggests addition, but subtraction is used to solve the problem.

> Daniel has 13 fish. Daniel has 4 more fish than Carlos. How many fish does Carlos have?

Help students understand that they should first represent the situation with a drawing and then decide how to solve the problem. Explain that using verbal cues, such as *more*, can lead them to use the wrong operation.

Identifying Needed Information

Lesson 6

Often real world problem situations do not have the exact information needed to solve a problem. A good problem solver has to be able to identify what information is needed to answer the question and to ignore any unnecessary information.

A good problem solver also has to recognize when he or she has to find more information in order to answer a question. Sometimes the information is hidden within the given information; for example, knowing that one mile is the same as 5,280 feet. Sometimes there is not enough information and the problem solver must decide whether to guess or estimate the missing information or just say that the problem cannot be solved without further information.

This lesson gives students practice with word problems that simulate such situations. Guide them to identify the information they need, translate any hidden information, and decide whether they have enough information to solve the problem.

Extra Information

2. Mark had 6 shirts ~~and 5 pairs of pants~~. Today his aunt gave him 4 more shirts ~~and another pair of pants~~. How many shirts does he have now?

10 shirts

Hidden Information

9. Lisa had 3 quarters and 2 dimes. Then she found 3 nickels and 12 pennies. What is the value of the coins in cents she has now?

122 cents

Not Enough Information

11. Sara bought 8 bananas at the fruit market. She put them in a bowl with some oranges. How many pieces of fruit are in the bowl?

You need to know how many oranges are in the bowl.

Two Step Word Problems

Lessons 7 8 9 10

Problems that involve two steps usually involve more than one operation. When this happens, the order in which the operations are performed is important. Students will apply what they learned earlier about using parentheses to show order of operations. The problems in these lessons involve all four operations.

First Step Questions Students begin their work with two step word problems by considering what question has to be answered in the first step. You might review First Step Questions from Unit 2.

Write the first step question and answer. Then solve the problem.

1. The orchard has 8 rows of apple trees. There are 7 rows with 6 apple trees and one row with 4 apple trees. How many apple trees are in the orchard?

   How many apples trees are in 7 rows? 42 trees; 46 trees

Use Equations In Lessons 8–10, students write equations to represent problems that are solved using two steps, and may need to include parentheses to indicate which operation is performed first. In their equations, students will use a letter to represent the unknown quantity. Encourage students to use drawings to represent the situation before they write the equations. This is good practice even for students who may be able to write an equation to represent the situation. Some students may use the drawings to help check their work.

Write an equation and solve the problem.

1. Mrs. Delgado is baking pies and cakes for a school fundraiser. She bought 26 apples, 29 peaches, and a number of bananas at the Farmers' Market. She bought 66 pieces of fruit. How many bananas did she buy?

   11 bananas; $26 + 29 + n = 66$, $n = 11$

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Using a letter for the unknown quantity, the order of operations, and two step word problems with all four operations As with two step problems at Grade 2, which involve only addition and subtraction, the Grade 3 two step word problems vary greatly in difficulty and ease of representation. More difficult problems may require two steps of representation and solution rather than one. Use of two step problems involving easy or middle difficulty adding and subtracting within 1,000 or one such adding or subtracting with one step of multiplication or division can help to maintain fluency with addition and subtraction while giving the needed time to the major Grade 3 multiplication and division standards.

Reasonable Answers An important part of performing computations and solving equations is checking exact answers for reasonableness.

Although students can check exact answers by performing computations a second time, they may more quickly and easily use number sense to assess reasonableness of answers. Students may use rounding by comparing an exact answer to a rounded answer or they may use mental math to find an approximate answer to compare with their calculated answer.

Use rounding or mental math to decide if the answer is reasonable. Write *yes* or *no*. Then write an equation and solve the problem to see if you were correct.

10. Chelsea's class collected cans of food for their local food pantry. They collected 27 cans on Monday and 78 cans on Tuesday. Then on Wednesday they collected 53 cans. How many cans did the class collect on those three days?
Answer: 158 cans

yes; $27 + 78 + 53 = 158$, 158 cans

Properties of Operations In Lesson 9, you will guide the students through an in-depth review of the properties of operations with a focus on how the properties can make computations easier. Encourage children to think strategically about a computation before they simply begin calculating.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Using a letter for the unknown quantity, the order of operations, and two step word problems with all four operations Understanding and using the Associative and Distributive Properties … requires students to know two conventions for reading an expression that has more than one operation:

1. Do the operation inside the parentheses before an operation outside the parentheses (the parentheses can be thought of as hands curved around the symbols and grouping them).

2. If a multiplication or division is written next to an addition or subtraction, imagine parentheses around the multiplication or division (it is done before these operations). At Grades 3 through 5, parentheses can usually be used for such cases so that fluency with this rule can wait until Grade 6.

Focus on Mathematical Practices

Lesson 11

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use what they know about operations with whole numbers and writing equations to solve problems related to interesting sports news items.

Little League Baseball Championships: Wheaton Wolves Score Win

Wheaton Wolves win Little League World Series Championship. The chart shows some statistics from the six games the team played.

Wheaton Wolves Statistics	
Times at Bat	155
Hits	47
Base on Balls	25
Runs Scored	36
Strike Outs	36