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Putting Research into Practice

From Current Research: Understanding Time

Most major difficulties in telling time are related to the efficient, but difficult analog clock.

- The same scale is used to tell the hour and the minutes, but different parts of the scale are used for hours and for minutes.
- There are two hands and two unit values (hours and minutes), and children must learn which hand is the hours and which hand is the minutes, which hand goes fast and which hand goes slow, which hand uses numbers and which hand uses five times the numbers, and which unit value is said first and shown first on a digital clock.
- The scale curves in a circle, which creates several difficulties:

—The beginning and end points of the scale are the same, and only the endpoint 12 is shown; there is no 0, so children start at 12 and go to 1, then 2, etc., which is not typical of how numbers usually go.

—The scale moves around in a circle, and children must learn the clockwise direction in which the numbers go if they are to visualize both hands going around.

- The use of 12 rather than 24 hours in the United States means that children must learn two sets of time, A.M. and P.M., and when these occur during the day.

Digital clocks are much easier, and children can begin to function with them as soon as they can read numbers to 60. The common practice of just reading the numbers without reading the units makes it easy for children to say the digital time (three-forty or nine-thirty-seven), but understanding what that means or how these numbers relate to an analog clock requires making many connections.

Fuson, Karen C., Douglas H., Beckman, Sybilla. *Focus in Grade 2: Teaching with Curriculum Focal Points,* National Council of Teachers of Mathematics, 2011. 83.

From Our Curriculum Research Project: Graphs

The goal of this unit is for children to describe, organize, and represent data in graphs. Children are introduced to the concept that some information, whether it be written or spoken, can be shown in a visual way. When presented as a graph, the information becomes readily accessible to children because they can study the relationships of the quantities depicted, either as parts of a whole or as separate entities to compare.

Children learn how to read picture graphs and bar graphs, and they use information from the graphs to answer questions and solve word problems. They also create their own picture graphs and bar graphs.

From Current Research: Statistics and Learning to Use Data

Processes like organizing data and conventions like labeling and scaling are crucial to data representation and are strongly connected to the concepts and processes of measurement. Given the difficulties students experience, instruction might need to differentiate these processes and conventions more sharply. Fundamental concepts… such as the conventions of scaling in graphs… need more careful attention in initial instruction.

National Research Council. "Developing Proficiency with Whole Numbers." *Adding It Up: Helping Students Learn Mathematics.* Washington, D.C.: National Academy Press, 2001. 288–294.

Another Useful Reference: Graphs

Jones, G.A., C.A. Thornton, C.W. Langrall, E.S Mooney, B. Perry, and I.J. Putt. "A Framework for Characterizing Children's Statistical Thinking." *Mathematical Thinking and Learning,* 2 (2000): 269–307.

Getting Ready To Teach Unit 5

Using the Common Core Standards for Mathematical Practice

The Common Core State Standards for Mathematical Content indicate what concepts, skills, and problem solving children should learn. The Common Core State Standards for Mathematical Practice indicate how children should demonstrate understanding. These *Mathematical Practices* are embedded directly into the Student and Teacher Editions for each unit in *Math Expressions*. As you use the teaching suggestions, you will automatically implement a teaching style that encourages children to demonstrate a thorough understanding of concepts, skills, and problems. In this program, Math Talk suggestions are a vehicle used to encourage discussion that supports all eight Mathematical Practices. See examples in Mathematical Practice 6.

COMMON CORE

Mathematical Practice 1
Make sense of problems and persevere in solving them.

Children analyze and make conjectures about how to solve a problem. They plan, monitor, and check their solutions. They determine if their answers are reasonable and can justify their reasoning.

TEACHER EDITION: Examples from Unit 5

MP.1 Make Sense of Problems Analyze Relationships Children work in **Helping Pairs** to complete comparison statements on Student Activity Book page 233. If children have difficulty completing the statements, ask, "Who has more?" After that is established, children may find it easier to determine how many more or how many fewer.
Invite children to use the graphs on Student Activity Book page 233 to make up their own questions for the rest of the class. Be sure children ask comparison questions using the words *more* and *fewer*. Remind them that the questions can be expressed both ways.

Lesson 4 ACTIVITY 1

MP.1 Make Sense of Problems Analyze the Problem Since these problems have more than one step, remind children to think about the hidden question in each problem

- **Problem 4:** How many purple beads does Morgan have? 10 purple beads
- **Problem 5:** How many red beads does Morgan have now? 8 red beads
- **Problem 6:** How many small beads does Morgan have in the beginning? 15 small beads

Lesson 7 ACTIVITY 1

Mathematical Practice 1 is integrated into Unit 5 in the following ways:

Make Sense of Problems	Make a Table	Analyze the Problem
Justify Reasoning	Draw a Diagram	Describe Relationships
Make a Graph	Analyze Relationships	

COMMON CORE

Mathematical Practice 2
Reason abstractly and quantitatively.

Children make sense of quantities and their relationships in problem situations. They can connect diagrams and equations for a given situation. Quantitative reasoning entails attending to the meaning of quantities.

TEACHER EDITION: Examples from Unit 5

MP.2 Reason Abstractly

- When you look at the clock, can you tell whether it is 6 o'clock in the morning or 6 o'clock in the evening? no

Remind children that the clock's hour hand moves in a complete circle twice each day. Tell them that we say A.M. for times after midnight and before noon (morning) and P.M. for times after noon and before midnight (afternoon and evening).

- What is an activity that you might be doing at 6:00 A.M.? Possible answer: waking up
- What is an activity that you might be doing at 6:00 P.M.? Possible answer: eating dinner

Refer children to Student Activity Book page 222. Ask them to complete Exercises 11–14 on their own and discuss their answers as a class.

Lesson 1 ACTIVITY 1

MP.2 Reason Abstractly and Quantitatively Draw a circle on the board. Now draw a vertical line segment to partition the circle into equal halves. Explain that the line segment divides the circle into two **equal shares**. Each share or part is **half** of the whole shape. The circle has two **halves**. Relate the equal halves of a circle to the half-hour.

Lesson 2 ACTIVITY 2

Mathematical Practice 2 is integrated into Unit 5 in the following ways:

Reason Abstractly
Reason Abstractly and Quantitatively

COMMON CORE

Mathematical Practice 3

Construct viable arguments and critique the reasoning of others.

Children use stated assumptions, definitions, and previously established results in constructing arguments. They are able to analyze situations and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.

Children are also able to distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Children can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MATH TALK is a conversation tool by which children formulate ideas and analyze responses and engage in discourse. See also MP.6 Attend to Precision.

TEACHER EDITION: Examples from Unit 5

MP.3, MP.6 Construct Viable Arguments/Critique the Reasoning of Others Puzzled Penguin Children help Puzzled Penguin tell the correct time on Student Activity Book page 230. Some children may mistakenly read 4:55 as 5:55 because the hour hand is so close to 5. Encourage children to explain that Puzzled Penguin should remember when the hour hand is between two numbers, the hour is always the lesser number.

Have children cross out the time Puzzled Penguin wrote and insert the correct time on the digital clock in Exercise 26.

Lesson 2 ACTIVITY 2

MP.3 Construct Viable Arguments Justify Conclusions Ask questions such as the following about the data in the graph.

- Is the number of bicycles sold about the same each day? Possible answer: No, more bicycles are sold on Saturday and Sunday than on Monday and Tuesday.
- How many fewer bicycles were sold on Monday and Tuesday than on Saturday and Sunday? 17 fewer bicycles

Use the questions and the graphs to help children choose which day the store should close. They should interpret the data to see that bicycle sales are higher over the weekend so the store should close on a Monday or Tuesday.

Lesson 9 ACTIVITY 1

Mathematical Practice 3 is integrated into Unit 5 in the following ways:

Critique the Reasoning of Others
Construct Viable Arguments
Justify Conclusions
Puzzled Penguin
Compare Representations

COMMON CORE

Mathematical Practice 4
Model with mathematics.

Children can apply the mathematics they know to solve problems that arise in everyday life. This might be as simple as writing an equation to solve a problem. Children might draw diagrams to lead them to a solution for a problem. Children apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation. They are able to identify important quantities in a practical situation and represent their relationships using such tools as diagrams, tables, graphs, and formulas.

TEACHER EDITION: Examples from Unit 5

MP.1, MP.4 Make Sense of Problems/ Model with Mathematics Make a Graph Give 8 pennies to one child (Maya), 5 pennies to another child (Anthony), and 2 pennies to a third child (Sona). Then invite the class to record this by making a graph on their MathBoards. Have children hold up their MathBoards so that their classmates can see them. Discuss which drawings best represent all the information given. (Children's graphs may vary depending upon their prior experience with graphing.)
Formalize what children have done on their own by making a picture graph as a way to represent information. Explain to the class that you will use a picture graph to show how many pennies each child has. Be sure that children label their own graphs as you draw the graph shown.

Lesson 3 ACTIVITY

MP.4 Model with Mathematics Draw a Diagram Begin collecting data. Limit the number of choices to four. You may need to use "Other" as one of the choices. The topic of favorite color is used here as an example. Ask each child to tell the class his or her favorite color. Record their answers using circles in 5-groups. As you record the data, remind children how to use circles in 5-groups.

Lesson 8 ACTIVITY 1

Mathematical Practice 4 is integrated into Unit 5 in the following ways:

- Model with Mathematics
- Draw a Diagram
- Make a Graph
- Make a Table

COMMON CORE

Mathematical Practice 5

Use appropriate tools strategically.

Children consider the available tools and models when solving mathematical problems. Children make sound decisions about when each of these tools might be helpful. These tools might include paper and pencil, a straightedge, a ruler, or the MathBoard. They recognize both the insight to be gained from using the tool and the tool's limitations. When making mathematical models, they are able to identify quantities in a practical situation and represent relationships using modeling tools such as diagrams, grid paper, tables, graphs, and equations.

Modeling numbers in problems and in computations is a central focus in *Math Expressions* lessons. Children learn and develop models to solve numerical problems and to model problem situations. Children continually use both kinds of modeling throughout the program.

TEACHER EDITION: Examples from Unit 5

MP.5 Use Appropriate Tools Analog Clock Have **Student Helpers** distribute scissors, prong fasteners, and paper plates to children and refer the class to Student Activity Book page 223. Have them use this page to create their own analog clock. Assist children as necessary, particularly in attaching the clock to the paper plate and then attaching the clock hands with the prong fastener. Tell children that they will be using their clocks in Daily Routines.
Children may need help remembering which hand moves quickly over the hour and which one moves slowly. Ask them to think of a fast animal and a slow animal and write the letter for that animal (or draw it quickly) on their clock's hour hand and minute hand.

Lesson 1 ACTIVITY 2

MP.5 Use Appropriate Tools MathBoard Demonstrate to children how to turn their picture graph into a bar graph. While you shade the squares that have pictures in them, have children do the same on their MathBoards. Have children explain why bar graphs are easier to make than picture graphs. You don't have to draw all the pictures.
You can use the MathBoard (the squares are 1 in. × 1 in.) and a ruler to show children how to place numbers on a bar graph. Place the ruler directly below the graph. Line up the zero mark with the left edge of the grid. Then slide the ruler down. Write the numbers at the bottom of the graph above the ruler. Be sure to include zero. Have children do the same on their graphs. Emphasize that bar graphs are like rulers; the numbers are placed under the vertical grid lines.

Lesson 5 ACTIVITY 1

Mathematical Practice 5 is integrated into Unit 5 in the following ways:

Use Appropriate Tools
MathBoard

Analog Clock

COMMON CORE

Mathematical Practice 6
Attend to precision.

Children try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. They are careful about specifying units of measure to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. Children give carefully formulated explanations to each other.

TEACHER EDITION: Examples from Unit 5

Ask Questions About Picture Graphs

WHOLE CLASS | MATH TALK

Invite volunteers to ask questions about information in the picture graph. Have the class answer the questions. Then invite children who are holding the pennies to rephrase the answers from their own point of view. See the examples of children's questions and responses below.

MP.6 Attend to Precision Explain a Representation

- Who has the most pennies? Maya
  Invite Maya to say, "I have more pennies than Anthony or Sona."
- Who has the fewest pennies? Sona
  Invite Sona to say, "I have the fewest pennies."
- How many more pennies does Maya have than Anthony? 3 more pennies
  Invite Maya to say, "I have 3 more pennies than Anthony."

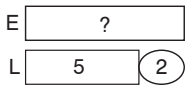
Lesson 3 ACTIVITY

MATH TALK in ACTION

Children discuss how to use comparison bars to help solve Problem 3.

Who would like to show comparison bars for Problem 3?

Jessie: Here is how I drew the bars.



Cara: How did you know to use the longer bar for elephants?

Jessie: Since there are fewer lions than elephants, there must be more elephants than lions. So the longer bar needs to go with elephants.

Manny: Where did the 2 and the 5 come from?

Jessie: It says in the problem that there are 2 fewer lions than elephants. I got the 5 by using the graph to find the number of lions. The graph does not give any number for elephants. That's what we are trying to find out.

Abigail: This problem is tricky. So, the comparison bars really help me. At first I thought I should subtract because the problem uses the word *fewer*. But the comparison bars show me I need to add 5 + 2 to find the answer.

Lesson 7 ACTIVITY 1

Mathematical Practice 6 is integrated into Unit 5 in the following ways:

Attend to Precision
Explain a Representation
Explain a Solution
Puzzled Penguin

COMMON CORE

Mathematical Practice 7
Look for structure.

Children analyze problems to discern a pattern or structure. They draw conclusions about the structure of the relationships they have identified.

TEACHER EDITION: Examples from Unit 5

MP.7 Look for Structure Identify Relationships Refer children to Student Activity Book page 227. Explain that the small numbers shown around the outside of the clock face tell the number of minutes after the hour.

- How many minutes are there between 12 and 1 on the clock face? 5 minutes
- How many minutes are there between 12 and 2 on the clock face? 10 minutes

Have children fill in the boxes with the number of minutes after the hour in 5-minute intervals.

When they are finished, ask them how many minutes are on the clock. 60 minutes

- How many minutes are in 1 hour? 60 minutes

Lesson 2 ACTIVITY 1

MP.7 Look for Structure Identify Relationships Direct children's attention to the bar graph at the top of Student Activity Book page 237. Sketch the same graph on the board or use a transparency of the page.

Point out the title, labels, and scale on the graph. Make sure children can read the information.

Encourage children to discuss how they can figure out how many coins of each kind there are.

- How many coins from the United States do I have? 10
- How do you know? We can look at the length of the bar.
- How many coins from Canada do I have? 4 coins

Have children write the number of coins next to each bar. Explain that this will make it easier to ask and answer questions about the graph.

Lesson 6 ACTIVITY

Mathematical Practice 7 is integrated into Unit 5 in the following ways:

Look for Structure
Identify Relationships

Use Structure

COMMON CORE

Mathematical Practice 8

Look for and express regularity in repeated reasoning.

Children use repeated reasoning as they analyze patterns, relationships, and calculations to generalize methods, rules, and shortcuts. As they work to solve a problem, children maintain oversight of the process while attending to the details. They continually evaluate the reasonableness of their intermediate results.

TEACHER EDITION: Examples from Unit 5

MP.8 Use Repeated Reasoning

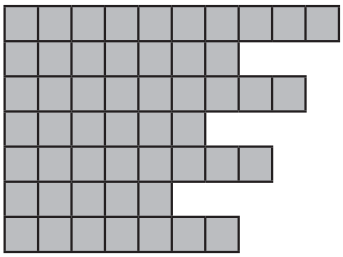
Generalize Help children connect the practice of counting by 5s to reading time to 5 minutes. Encourage children to practice counting by 5s by reading the minutes on a clock.

Write 5, 10, 15, and so on around the outside of the clock on the Time Poster as shown. The class says the minute numbers as you point to them.

Lesson 2 ACTIVITY 1

MP.8 Use Repeated Reasoning

Draw this pattern sequence on the board.



Children can explain any patterns they see.

Lesson 10 ACTIVITY 2

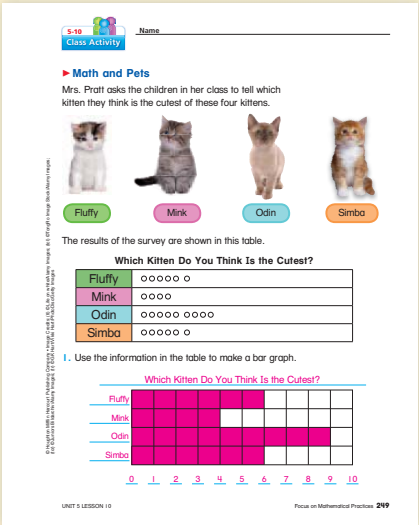
Mathematical Practice 8 is integrated into Unit 5 in the following ways:

Use Repeated Reasoning | **Draw Conclusions** | **Generalize**

FOCUS on Mathematical Practices

Unit 5 includes a special lesson that involves solving real world problems and incorporates all 8 Mathematical Practices. In this lesson, children collect, organize, and graph data.

STUDENT EDITION: LESSON 10, PAGES 249–250



5-10 Class Activity Name

Math and Pets

Mrs. Pratt asks the children in her class to tell which kitten they think is the cutest of these four kittens.

Fluffy | Mink | Odin | Simba

The results of the survey are shown in this table.

Which Kitten Do You Think Is the Cutest?

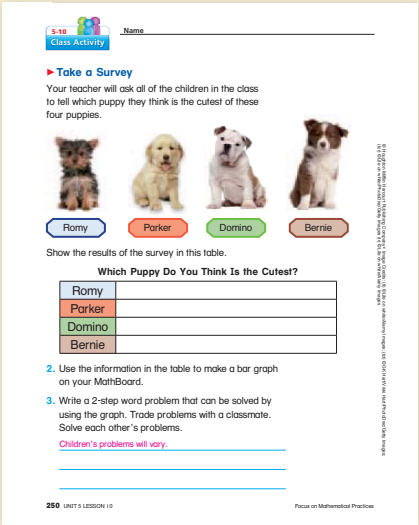
Fluffy	○○○○○ ○
Mink	○○○○
Odin	○○○○○ ○○○○
Simba	○○○○○ ○

1. Use the information in the table to make a bar graph.

Which Kitten Do You Think Is the Cutest?

Fluffy, Mink, Odin, Simba; 0 1 2 3 4 5 6 7 8 9 10

UNIT 5 LESSON 10 Focus on Mathematical Practices 249



5-10 Class Activity Name

Take a Survey

Your teacher will ask all of the children in the class to tell which puppy they think is the cutest of these four puppies.

Romy | Parker | Domino | Bernie

Show the results of the survey in this table.

Which Puppy Do You Think Is the Cutest?

Romy	
Parker	
Domino	
Bernie	

2. Use the information in the table to make a bar graph on your MathBoard.
3. Write a 2-step word problem that can be solved by using the graph. Trade problems with a classmate. Solve each other's problems.

Children's problems will vary.

250 UNIT 5 LESSON 10 Focus on Mathematical Practices

Getting Ready to Teach Unit 5

Learning Path in the Common Core Standards

In this unit, children will read and show time to the 5 minutes, display data in bar graphs and picture graphs, and interpret the data in graphs to solve problems.

Visual models and real world situations are used throughout the unit to help children understand the concepts of time and graphing.

Help Children Avoid Common Errors

Math Expressions gives children opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit we use Puzzled Penguin to show typical errors that children make. Children enjoy teaching Puzzled Penguin the correct way, why this way is correct, and why Puzzled Penguin made the error. Common errors are presented in the Puzzled Penguin feature in the following lessons:

- **Lesson 2:** Choosing the incorrect hour when the hour hand is closer to the greater number
- **Lesson 4:** Reading the data in a picture graph incorrectly when the pictures are not aligned
- **Lesson 7:** Not realizing that a problem has more than one step
- **Lesson 8:** Not beginning the scale on a bar graph at 0.

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

***Math Expressions* VOCABULARY**

As you teach this unit, emphasize understanding of these terms.

- 5-group circles
- comparison bars

See the Teacher Glossary.

Working with Time

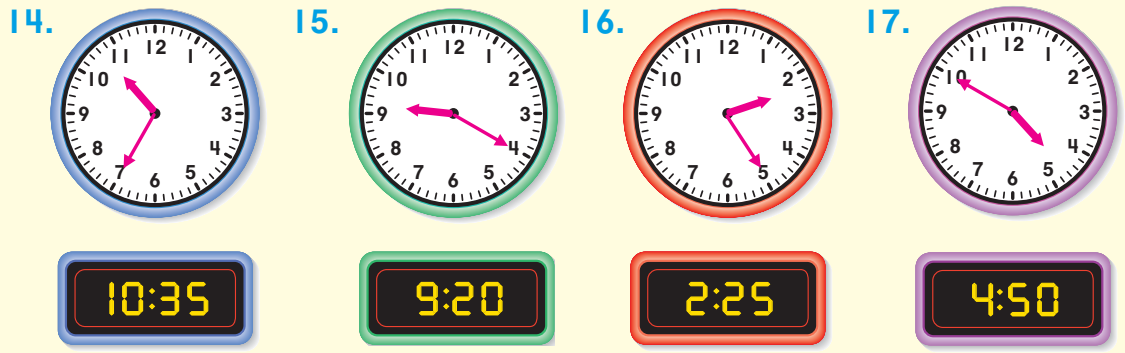
Lessons 1 2

The Grade 2 Common Core State Standards for Measurement and Data require children to tell and write time from analog and digital clocks to the nearest five minutes, using A.M. and P.M.

Time and Clocks Children work with the two kinds of clocks that are most often used for telling time—analog and digital. Knowing how to tell time with both kinds of clocks is an important real world skill as so much of what we do is driven by time—appointments, schedules, planning trips, and much else.

As they work with time in Grade 2, children build an analog clock that they use to tell and show times. They also draw hands on analog clock faces and write times in digital clock images.

Draw hands on each clock to show the time.

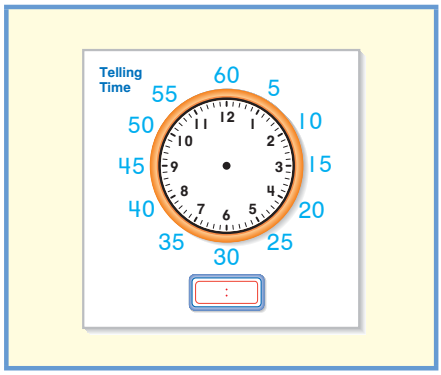


Understanding how analog clocks work helps children to understand that a clock is a tool for measuring time with iterated units, just as a ruler is a tool for measuring length with iterated units. As in the customary length system a unit, the inch, is repeated to measure lengths and to form larger units, such as a foot or a yard or a mile, so in the system of time a unit, the minute, is repeated to measure time and to form larger units, such as an hour or a day or a year.

Guide children to observe that as the hour hand moves from one number to another, the minute hand moves all around the clock face. Some children may observe that to do this, the minute hand moves faster than the hour hand.

Connection to Geometry As children work with telling time to the half-hour, it may be helpful for them to relate two halves of an hour to two halves of a circle. When the minute hand is at 6 or 30 minutes, it is halfway through an hour and it has moved halfway around the clock face. Some children may also notice that the hour hand has moved halfway between the two hour numbers.

Skip Counting to Find Time Amounts To help children make the connection between the numbers on an analog clock and the number of minutes they represent, they use skip counting by 5s. They count by five-minute intervals as the clock numbers are pointed out on the Time Poster, which you have labeled with the numbers of minutes.



Daily Routine for Time Use this new daily routine every day as you teach this unit to provide practice with telling time to 5 minutes and understanding differences between A.M. and P.M. Five Student Leaders are needed to help with this routine.

The tasks are counting by fives around the clock with the Time Poster, telling digital time to 5 minutes, and showing clock face time to 5 minutes. Children use the clocks they made in Lesson 1 to show their responses.

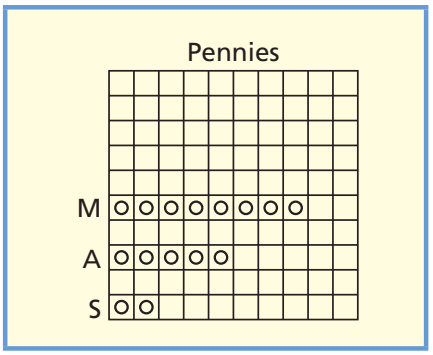
from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Understand Place Value Students begin to work towards multiplication when they skip count by 5s, by 10s, and by 100s. This skip counting is not yet true multiplication because students don't keep track of the number of groups they have counted.

Picture Graphs

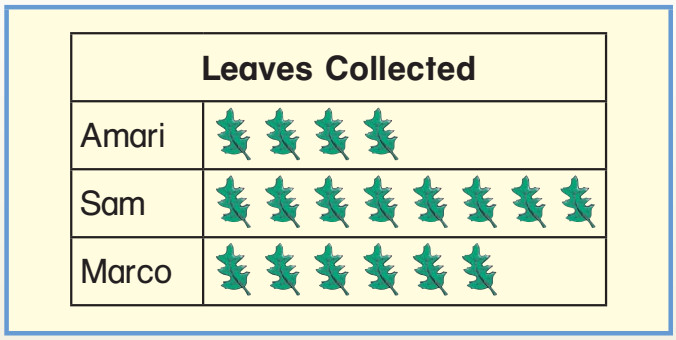
Lessons 3 4

Make a Picture Graph Children construct their first picture graph by using the 10 × 10 Grid on their MathBoards to represent a situation in which you give each of three children a different numbers of pennies. After children work independently to record this information in a graph, they share and discuss their work. Then you guide them through a more formal process of making and labeling a picture graph. Point out that drawing circles to represent the pennies is easier and quicker than trying to draw pictures of the pennies.



Ask Questions Help children understand that collecting and representing data is only part of learning to work with data. Explain another important part of working with data using a graph is learning about the data by examining the graph. Guide children to ask and answer questions about the data in the graph.

Solve Problems Children use picture graphs shown in the Student Activity Book to solve problems and then make up their own questions about those graphs. Help them focus on asking comparative and equalizing questions about the data.



Asking questions and solving problems about picture graphs prepares children for similar work with bar graphs.

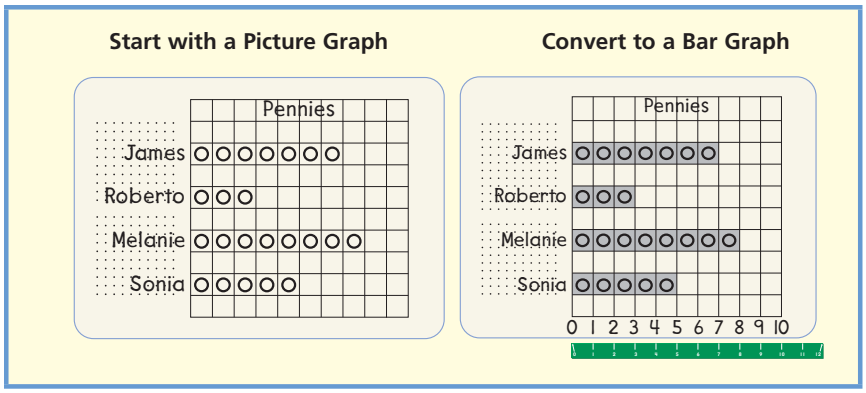
from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON MEASUREMENT AND DATA

Categorical Data Students in Grade 2 draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories.

Bar Graphs

Lessons 5 6 7 8 9 10

Make a Bar Graph Converting a picture graph to a bar graph helps children understand what the bars in a bar graph represent. Children begin by making a picture graph on the 10 × 10 Grid on their MathBoards.



Children see that the length of each bar represents the total number of things (in that category). Point out that in a bar graph the length of each bar shows the total number of that item, but in a picture graph, each item is shown separately so that the total number has to be counted.

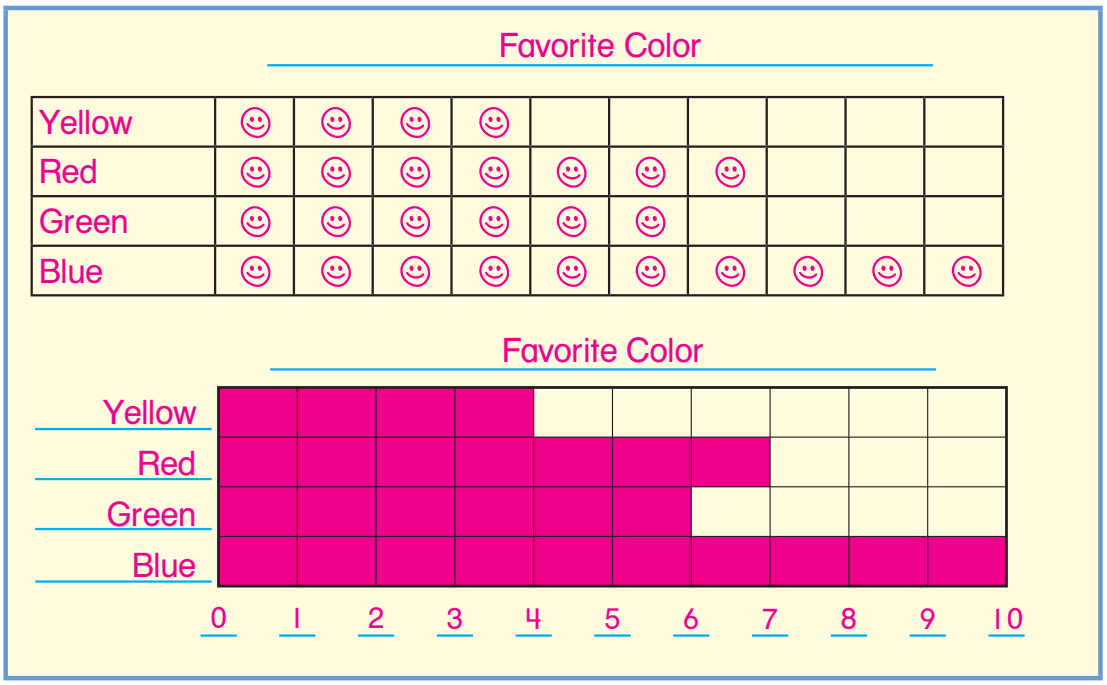
To support the idea that a bar graph is a length model, children use an inch ruler to label the bottom axis with the count scale. Be sure that children place the numbers under the vertical lines on the grid rather than centering them in the spaces of the grid and align the 0 on the ruler with the left edge of the grid.

Horizontal and Vertical Bars In this unit, children will work with bar graphs that have both horizontal bars and vertical bars. To help them understand the differences between the two kinds of bar graphs, they will convert a horizontal bar graph to a vertical bar graph in Lesson 6. As they do this, they discuss how and why the scale moves from the horizontal axis to the vertical axis. Children should also note that they do not change the scale and that in the vertical bar graph, each vertical bar representing a quantity corresponds to a horizontal bar representing that quantity in the horizontal bar graph.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON MEASUREMENT AND DATA

Categorical Data When drawing bar graphs on grid paper, the tick marks on the count scale should be drawn at intersections of the gridlines. The tops of the bars should reach the respective gridlines of the appropriate tick marks. When drawing picture graphs on grid paper, the pictures representing the objects should be drawn in the squares of the grid paper.

Collect and Organize Data Children learn to use a table to organize data before making a graph. They later use a simple survey to collect data about a topic of their choice, such as favorite color, food, or game. They organize the data they collect in a table and then make both a picture graph and a bar graph.



Solve Problems Children solve *Put Together/Take Apart* and *Compare* story problems using information presented in bar graphs shown in the Student Activity Book. They also make up their own questions about graphs in the Student Activity Book and their own graphs. Guide them especially to ask comparative questions about the data. When children make a comparison, ask them to present it from both points of view.

> Three more children liked red better than yellow.
>
> Three fewer children liked yellow better than red.

As children gain experience with using bar graphs to solve problems, they will notice that they can simply look at the bars in a graph to make a quick comparison and then find the length of the bars to make a more precise comparison.

***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON MEASUREMENT AND DATA**

Categorical Data Students in Grade 2 … solve simple put-together, take-apart, and compare problems using information presented in a bar graph.