Getting Ready to Teach Unit 2

Learning Path in the Common Core Standards

In Unit 2, children begin to recognize addition and subtraction problem types and write equations to represent addition and subtraction situations. Equations are emphasized as children discuss different types of equations, decide if they are true or false, and relate addition and subtraction equations. They develop strategies for adding and subtracting within 10.

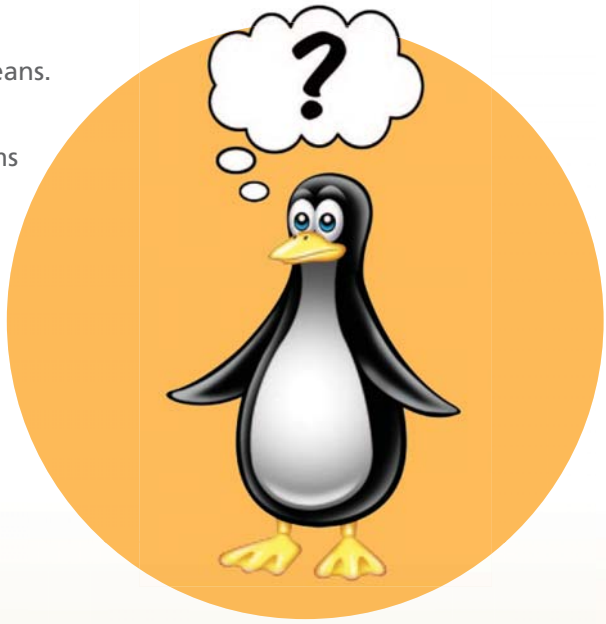
Help Children Avoid Common Errors

Math Expressions gives children opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit we use Puzzled Penguin to show typical errors that children make. Children enjoy teaching Puzzled Penguin the correct way, why this way is correct, and why Puzzled Penguin made an error. Common errors are presented in the Puzzled Penguin feature in the following lessons:

- **Lesson 4:** Does not understand what the equal sign means.
- **Lesson 6:** Incorrectly counts on to add.
- **Lesson 10:** In a subtraction picture, thinks that the items crossed out represent the answer.
- **Lesson 14:** Does not pay attention to the symbol, and adds instead of subtracting.

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.



***Math Expressions* VOCABULARY**

As you teach this unit, emphasize understanding of these terms.

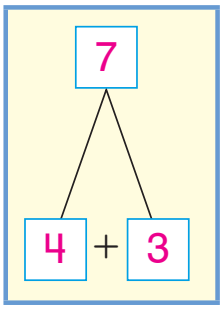
- circle drawing
- partner
- Proof Drawing

See the Teacher Glossary.

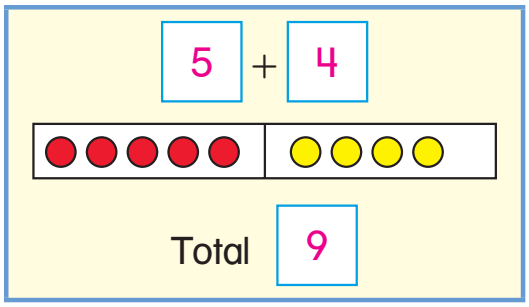
Representing Addition

Lessons 1 2

Partners and Totals The models used are designed to help children make the transition to equations in different forms. Children begin by writing partners and totals (expressions) using pictures and then connecting them to Math Mountains.



With Math Mountains, the total is on top, leading children to equations with the total first, such as 7 = 4 + 3.

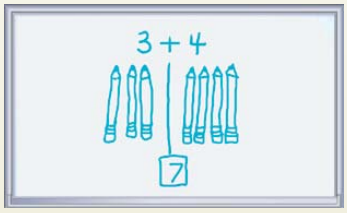


Children are then introduced to circle drawings and equations where the partners (addends) come first, leading them to equations such as 5 + 4 = 9.

Story Problems Children represent story problems throughout the unit.

First they draw pictures to represent the problem. Initially, they draw each item in the problem.

I buy 3 pencils. You buy 4 pencils. How many do we buy altogether?



Children progress rapidly to circle drawings. Representing the problem with circle drawings simplifies the process from drawing real objects.

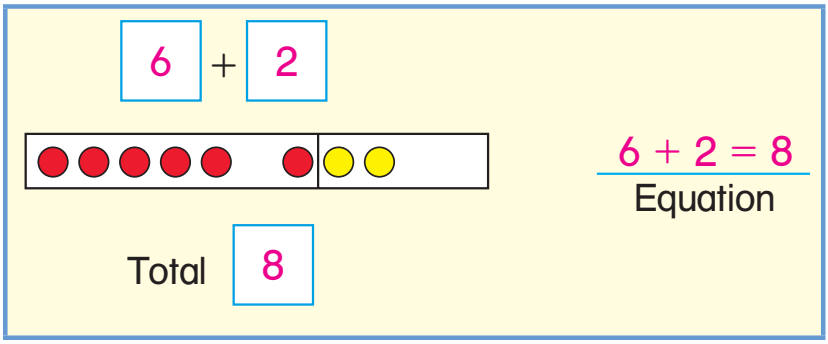
from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Both Addends Unknown Equations with one number on the left and an operation on the right (e.g., 5 = 2 + 3 to record a group of 5 things decomposed as a group of 2 things and a group of 3 things) allow students to understand equations as showing in various ways that the quantities on both sides have the same value.

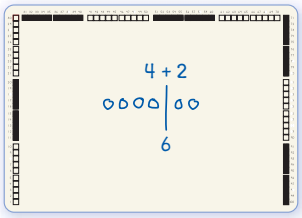
Understanding Equality

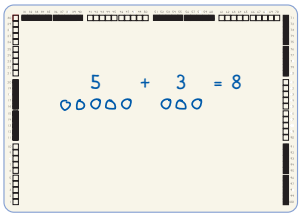
Lessons 3 4

Writing Equations Children connect writing partners and totals to writing equations.

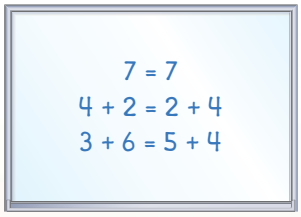


They also discuss whether two quantities are equal or not equal and whether an equation is true or false. They make Proof Drawings to show that an equation is true.





They also discuss other types of equations.



These discussions emphasize the meaning of equality. As they compare both the *equal* sign and the *not equal* sign, children can avoid the common error of thinking of the equal sign as punctuation before or after the answer rather than a symbol meaning "is the same amount or value."

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

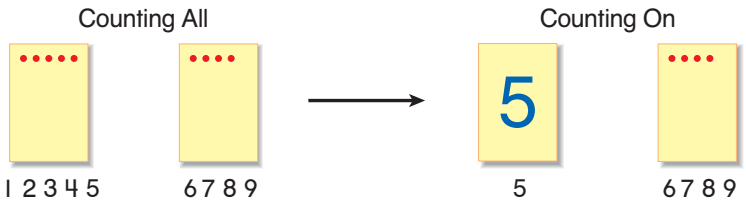
Put Together/Take Apart Situations
Two quantities jointly compose a third quantity (the total), or a quantity can be decomposed into two quantities (the addends). This composition/decomposition may be physical or conceptual. These situations are acted out with objects initially and later children begin to move to conceptual mental actions of shifting between seeing the addends and seeing the total (e.g., seeing children or seeing boys and girls, or seeing red and green apples or all the apples).

Counting On to Add

Lessons 5 6 7 8 9

Children's Strategies

Compare Counting Strategies Children are introduced to comparing "counting all" to "counting on." *Counting on* differs from *counting all* in that the counting is abbreviated by counting on from the greater number. This is especially important when children start adding numbers with totals greater than 10.



Count On with Fingers Then children learn how to use their fingers to count on to find a total. The child is monitoring the known partner to decide when to stop counting.

I have 5.

Count On with Dots Children also learn how to draw dots to represent one addend. Then they start with one partner and count the dots to find the total.

5 + 4 = ☐

5 ••••

5 + 4 = 9

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

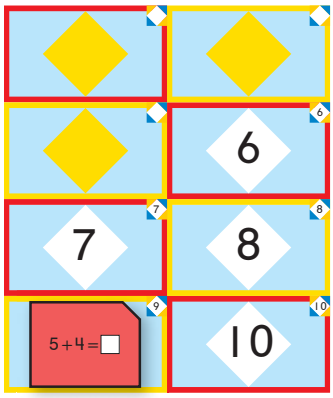
Level 2 Counting On Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. Some method of keeping track (fingers, objects, mentally imaged objects, body motions, other count words) is used to monitor the count. For addition, the count is stopped when the amount of the remaining addend has been counted. The last number word is the total. For subtraction, the count is stopped when the total occurs in the count. The tracking method indicates the difference (seen as an unknown addend).

Commutative Property Children are reminded that they can add two numbers in any order—they can switch the partners. Then they count on from the greater number. They confirm that the total is the same either way.

$$3 + \underline{7} = \boxed{10}$$

Games and Scenarios to Build Fluency Games and real world scenarios are designed to reinforce the Counting On Strategy and to help children work toward fluency.

The Number Quilt Game



Number Quilt 1: Unknown Totals

How to Play Children read the equation on the first card and count on to find the total. Then they check the answer on the back. If correct, they place the card on the quilt in the section that corresponds to the total. The small numbers in the corners allow children to continue placing cards even after the large central numbers have been covered. If they are not correct, the card goes back in the pile to use again.

The Pancake Breakfast

Children pretend to make pancakes. Each child counts aloud as he or she takes plates from the stack.

- First child: "I am making 1, 2, 3, 4 pancakes."
- Second child: "I am making 1, 2, 3, 4, 5, 6 pancakes."

A volunteer writes the equation on the board.

$$4 + 6 = \square$$

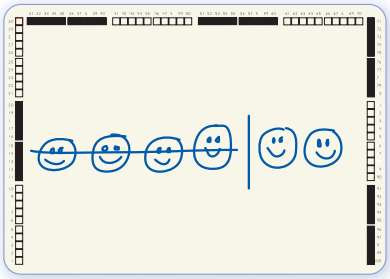
Children count on from the greater number to find the total. When the class has agreed on the total, the "pancake makers" line up the plates on the ledge of the board so the class can check the answer.

Representing Subtraction

Lessons 10 11 12 13

Subtraction with Pictures Subtraction problems begin in a way similar to addition. For subtraction, however, the objects being subtracted are crossed off after being separated with a Break-Apart Stick.

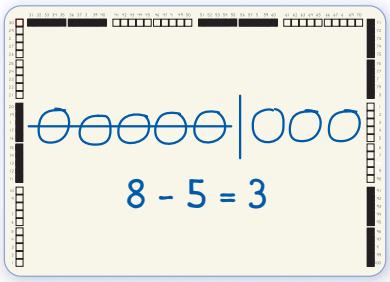
There are 6 people swimming.
Four of them go home.
Two people are still swimming.



In *Math Expressions*, children learn to take away the first items in subtraction to support counting on for subtraction, which will be taught in Unit 3. They are taught to use a line to cross off objects. This line mirrors a minus sign, which indicates subtraction.

Subtraction with Circle Drawings As with addition, children progress to circle drawings and label the various parts in subtraction problems. The unknown will always be the part that remains.

We have 8 apples.
Then we eat 5 of them.
How many apples do we have now?



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Meaning of Subtraction Addition and subtraction are the first operations studied. Initially, the meaning of addition is separate from the meaning of subtraction, and students build relationships between addition and subtraction over time. Subtraction comes to be understood as reversing the actions involved in addition and as finding an unknown addend.

Relating Addition and Subtraction

Lessons 14 15

Describe Relationships Children discuss how each shaded section relates to the story problem and describe the relationships they see. This is the introduction for relating addition and subtraction.

1. Addition	2. Subtraction
5 cats are here.	8 cats are here.
3 cats come.	5 cats run away.
How many cats in all?	How many cats are left?
5 + 3 = ●●●●● | ○○○	8 − 5 = ~~●●●●●~~ ○○○
5 + 3 = □	8 − 5 = □
5 + 3 = 8	8 − 5 = 3
Total Unknown	Partner Unknown
	5 + □ = 8

This leads children to practice sets for relating operations.

4 + 4 = 8, so I know 8 − 4 = 4.

Ultimately children practice solving related equations and vertical forms.

Equations	Vertical Forms	
5 + 3 = 8	5	8
8 − 5 = 3	+ 3	− 5
	8	3

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Algebraic Perspective Students thus begin developing an algebraic perspective many years before they will use formal algebraic symbols and methods. They read to understand the problem situation, represent the situation and its quantitative relationships with expressions and equations, and then manipulate that representation if necessary, using properties of operations and/or relationships between operations. Linking equations to concrete materials, drawings, and other representations of problem situations affords deep and flexible understandings of these building blocks of algebra.

Problem Solving Situations

Lessons 1 2 3 4 10 11 12 13 14 15 16

Put Together/Take Apart with Total Unknown In these problem types, two quantities compose a third quantity (the total), or a quantity can be decomposed into two quantities (the partners, or addends). Children learn to shift between seeing the partners and seeing the total (for example, seeing fruit or seeing apples and oranges, or seeing yellow and red flowers or all the flowers).

I have 6 white shirts and 3 black shirts.
How many shirts do I have in all?
$6 + 3 = 9$; 9 shirts

Add To/Take From with Result Unknown The relationship between addition and subtraction in these problem types is that of reversibility of actions: an Add To situation undoes a Take From situation and vice versa.

I have 3 teddy bears.
Then I get 4 more.
How many teddy bears do I have now?
$3 + 4 = 7$; 7 teddy bears

Brent buys a bag with 10 peanuts in it.
He eats 6 of them.
How many peanuts are left?
$10 - 6 = 4$; 4 peanuts

Focus on Mathematical Practices

Lesson 16

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, children use what they know about addition and subtraction to solve problems about an animal park.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Add To/Take From Problems These are action oriented problems that show changes from an initial state to a final state. These situations are readily modeled by equations because each aspect of the situation has a representation as number, operation (+ or −), or equal sign (=).

Math Talk Learning Community

Research In the NSF research project that led to the development of *Math Expressions*, much work was done with helping teachers and children build learning communities within their classrooms. An important aspect of doing this is Math Talk. The researchers found three levels of Math Talk that go beyond the usual classroom routine of children simply solving problems and giving answers and the teacher asking questions and offering explanations.

Math Talk Level 1 A child briefly explains his or her thinking to others. The teacher helps children listen to and help others, models fuller explaining and questioning by others, and briefly probes and extends children's ideas.

Example Story Problem: 6 birds are in the yard. 1 more bird joins them. How many birds are in the yard now?

Who can tell us how many birds are in the yard now?

Billy: There are 7 birds.

How do you know?

Billy: I know that 6 and 1 more is 7.

Who found a different way to answer the question?

Lucy: I made a drawing with 6 circles and 1 circle, and then counted on to find 7.

Math Talk Level 2 A child gives a fuller explanation and answers questions from other children. The teacher helps children listen and ask good questions, models full explaining and questioning (especially for new topics), and probes more deeply to help children compare and contrast methods.

Example Story Problem: Snow has 8 marbles and 2 boxes. How many marbles can she put in each box?

How can we find the answer to this problem?

Jake: Is this a problem that has more than one answer?

Why do you ask that, Jake?

Jake: Because I know more than one way to break apart 8.

Ruth: If we break apart 8, we can make a list of the ways. Do you think that is what we are supposed to do?

Nancy: I think so, because we have been learning about break-aparts and partner equations.

Ruth: And it does not say the boxes have to be the same.

What is one way to start the list?

Caleb: Let's start by writing the 8-partners 1 and 7, and then change by 1 more each time.

Nancy: I agree with Caleb.

Math Talk Level 3 The explaining child manages the questioning and justifying. Children assist each other in understanding and correcting errors and in explaining more fully. The teacher monitors and assists and extends only as needed.

Example Story Problem: Joe eats 5 green grapes and 4 purple grapes. How many grapes does he eat?

Who will show us how to find the answer?

Julia: I know that green grapes and purple grapes are both grapes, so I have to add 5 + 4. I know that is 9, so the answer is 9 grapes. I also made a drawing to be sure I was right. Here is my drawing.

Bob: I think your answer is right, but your drawing only shows 8. You need to fix your drawing.

Nancy: Yes, when I count your circles, you are showing 4 + 4. So draw another circle.

How can we be sure that we make the right drawing?

Julia: I should have checked my drawing to be sure that I made 5 + 4. So we should always check what we do.

Nancy: We need to count each thing when we make drawings. We have to be sure and not guess that it looks right.

Summary Math Talk is important not only for discussing solutions to story problems but also for any kind of mathematical thinking children do, such as explaining why each number in the count sequence is 1 more than the number before it, or how to use a drawing to subtract, or how to put two triangles together to make a rectangle.